Office of Financial Research
Working Paper #0001
January 5, 2012

A Survey of Systemic Risk Analytics

Dimitrios Bisias[1]
Mark Flood[2]
Andrew W. Lo[3]
Stavros Valavanis[4]

[1] MIT Operations Research Center
[2] Senior Policy Advisor, OFR, mark.flood@treasury.gov
[3] MIT Sloan School of Management, alo@mit.edu
[4] MIT Laboratory for Financial Engineering

www.treasury.gov/ofr

A Survey of
Systemic Risk Analytics[*]

Dimitrios Bisias[†] Mark Flood[‡] Andrew W. Lo[§] Stavros Valavanis[¶]

This Draft: January 5, 2012

We provide a survey of 31 quantitative measures of systemic risk in the economics and finance literature, chosen to span key themes and issues in systemic risk measurement and management. We motivate these measures from the supervisory, research, and data perspectives in the main text, and present concise definitions of each risk measure—including required inputs, expected outputs, and data requirements—in an extensive appendix. To encourage experimentation and innovation among as broad an audience as possible, we have developed open-source Matlab code for most of the analytics surveyed, which can be accessed through the Office of Financial Research (OFR) at `http://www.treasury.gov/ofr`.

Keywords: Systemic Risk; Financial Institutions; Liquidity; Financial Crises; Risk Management

JEL Classification: G12, G29, C51

[†]MIT Operations Research Center.

[‡]Office of Financial Research.

[§]MIT Sloan School of Management, MIT Laboratory for Financial Engineering, and AlphaSimplex Group, LLC.

[¶]MIT Laboratory for Financial Engineering.

Contents

1 Introduction

In July 2010, the U.S. Congress enacted the *Dodd Frank Wall Street Reform and Consumer Protection Act* (Dodd Frank Act), the most comprehensive financial reform bill since the 1930s. Among other things, the Dodd Frank Act created the Financial Stability Oversight Council (FSOC) and Office of Financial Research (OFR). The FSOC has three broad mandates: (1) to identify risks to financial stability arising from events or activities of large financial firms or elsewhere; (2) to promote market discipline by eliminating participants' expectations of possible government bailouts; and (3) to respond to emerging threats to the stability of the financial system.[1] The starting point for all of these directives is the accurate and timely measurement of systemic risk. The truism that "one cannot manage what one does not measure" is especially compelling for financial stability since policymakers, regulators, academics, and practitioners have yet to reach a consensus on how to define "systemic risk". While regulators sometimes apply Justice Potter Stewart's definition of pornography, i.e., systemic risk may be hard to define but they know it when they see it, such a vague and subjective approach is not particularly useful for measurement and analysis, a pre-requisite for addressing threats to financial stability.

One definition of systemic risk is "any set of circumstances that threatens the stability of or public confidence in the financial system" (Billio, Getmansky, Lo, and Pelizzon, 2010). The European Central Bank (ECB) (2010) defines it as a risk of financial instability "so widespread that it impairs the functioning of a financial system to the point where economic growth and welfare suffer materially". Others have focused on more specific mechanisms, including imbalances (Caballero, 2009), correlated exposures (Acharya, Pedersen, Philippon, and Richardson, 2010), spillovers to the real economy (Group of Ten, 2001), information disruptions (Mishkin, 2007), feedback behavior (Kapadia, Drehmann, Elliott, and Sterne, 2009), asset bubbles (Rosengren, 2010), contagion (Moussa, 2011), and negative externalities (Financial Stability Board, 2009).

This partial listing of possible definitions suggests that more than one risk measure will be needed to capture the complex and adaptive nature of the financial system. Because systemic risk is not yet fully understood, measurement is obviously challenging, with many competing—and sometimes contradictory—definitions of threats to financial stability. More-

---

[1] See Section §112(a)(1) (Pub.L. 111-203, H.R. 4173). The full range of detailed mandates, constraints, and authorities for the FSOC and OFR are covered in Sections §112–156 of the Act.

over, a single consensus measure of systemic risk may neither be possible nor desirable, as such a "Maginot" strategy invites a blindsided surprise from some unforeseen or newly emerging crisis mechanism. Instead, a robust framework for monitoring and managing financial stability must incorporate both a diversity of perspectives and a continuous process for re-evaluating the evolving structure of the financial system and adapting systemic risk measures to these changes. At the same time, to be useful in *measuring* systemic risk, a practical implementation must translate economic concepts into very particular choices: one must decide which attributes of which entities will be measured, how frequently and over what observation interval, and with what levels of granularity and accuracy. Summary measures involve further choices on how to filter, transform, and aggregate the raw inputs.

In this paper, we take on this challenge by surveying the systemic risk measures and conceptual frameworks that have been developed over the past several years, and providing open-source software implementation (in Matlab) of each of the analytics we include in our survey. These measures are listed in Table 1, loosely grouped by the type of data they require, and described in detail in Appendixes A–F. The taxonomy of Table 1 lists the analytics roughly in increasing order of the level of detail for the data required to implement them. This categorization is obviously most relevant for the regulatory agencies that will be using these analytics, but is also relevant to industry participants who will need to supply such data.[2] For each of these analytics, Appendixes A–F contain a concise description of its definition, its motivation, the required inputs, the outputs, and a brief summary of empirical findings if any. For convenience, in Appendix G we list the program headers for all the Matlab functions provided.

Thanks to the overwhelming academic and regulatory response to the Financial Crisis of 2007–2009, we face an embarrassment of riches with respect to systemic risk analytics. The size and complexity of the financial system imply a diversity of legal and institutional constraints, market practices, participant characteristics, and exogenous factors driving the system at any given time. Accordingly, there is a corresponding diversity of models and measures that emphasize different aspects of systemic risk. These differences matter. For

---

[2]An obvious alternate taxonomy is the venerable *Journal of Economic Literature* (JEL) classification system or the closely related EconLit taxonomy. However, these groupings do not provide sufficient resolution within the narrow subdomain of systemic risk measurement to be useful for our purposes. Borio and Drehmann (2009b) suggest a three-dimensional taxonomy, involving forecasting effectiveness, endogeneity of risks, and the level of structural detail involved. Those three aspects are reflected in the taxonomies we propose in this paper.

Systemic Risk Measure	Section
Macroeconomic Measures:	
Costly Asset-Price Boom/Bust Cycles	A.1
Property-Price, Equity-Price, and Credit-Gap Indicators	A.2
Macroprudential Regulation	A.3
Granular Foundations and Network Measures:	
The Default Intensity Model	B.1
Network Analysis and Systemic Financial Linkages	B.2
Simulating a Credit Scenario	B.3
Simulating a Credit-and-Funding-Shock Scenario	B.4
Granger-Causality Networks	B.5
Bank Funding Risk and Shock Transmission	B.6
Mark-to-Market Accounting and Liquidity Pricing	B.7
Forward-Looking Risk Measures:	
Contingent Claims Analysis	C.1
Mahalanobis Distance	C.2
The Option iPoD	C.3
Multivariate Density Estimators	C.4
Simulating the Housing Sector	C.5
Consumer Credit	C.6
Principal Components Analysis	C.7
Stress-Test Measures:	
GDP Stress Tests	D.1
Lessons from the SCAP	D.2
A 10-by-10-by-10 Approach	D.3
Cross-Sectional Measures:	
CoVaR	E.1
Distressed Insurance Premium	E.2
Co-Risk	E.3
Marginal and Systemic Expected Shortfall	E.4
Measures of Illiquidity and Insolvency:	
Risk Topography	F.1
The Leverage Cycle	F.2
Noise as Information for Illiquidity	F.3
Crowded Trades in Currency Funds	F.4
Equity Market Illiquidity	F.5
Serial Correlation and Illiquidity in Hedge Fund Returns	F.6
Broader Hedge-Fund-Based Systemic Risk Measures	F.7

Table 1: Taxonomy of systemic risk measures by data requirements.

example, many of the approaches surveyed in this article assume that systemic risk arises endogenously within the financial system. If correct, this implies that there should be measurable intertemporal patterns in systemic stability that might form the basis for early detection and remediation. In contrast, if the financial system is simply vulnerable to exogenous shocks that arrive unpredictably, then other types of policy responses are called for. The relative infrequency with which systemic shocks occur make it all the more challenging to develop useful empirical and statistical intuition for financial crises.[3]

Unlike typical academic surveys, we do not attempt to be exhaustive in our breadth.[4] Instead, our focus is squarely on the needs of regulators and policymakers, who, for a variety of reasons—including the public-goods aspects of financial stability and the requirement that certain data be kept confidential—are solely charged with the responsibility of ensuring financial stability from day to day. We recognize that the most useful measures of systemic risk may be ones that have yet to be tried because they require proprietary data only regulators can obtain. Nevertheless, since most academics do not have access to such data, we chose to start with those analytics that could be most easily estimated so as to quicken the pace of experimentation and innovation.

While each of the approaches surveyed in this paper is meant to capture a specific challenge to financial stability, we remain agnostic at this stage about what is knowable. The system to be measured is highly complex, and so far, no systemic risk measure has been tested "out of sample", i.e., outside the recent crisis. Indeed, some of the conceptual frameworks that we review are still in their infancy and have yet to be applied. Moreover, even if an exhaustive overview of the systemic risk literature were possible, it would likely be out of date as soon as it was written.

Instead, our intention is to present a diverse range of methodologies, data sources, levels of data frequency and granularity, and industrial coverage. We wish to span the space of what has already been developed, to provide the broadest possible audience with a sense of where the boundaries of the field lie today, and without clouding the judgments of that

---

[3]Borio and Drehmann (2009a) observe that there is as yet no single consensus explanation for the behavior of the financial system during crises, and because they are infrequent events in the most developed financial centers, the identification of stable and reliable patterns across episodes is virtually impossible in one lifetime. Caruana (2010a) notes two studies indicating that, worldwide, there are roughly 3 or 4 financial crises per year on average. Most of these have occurred in developing economies, perhaps only because smaller countries are more numerous.

[4]Other surveys are provided by Acharya, Pedersen, Philippon, and Richardson (2010), De Bandt and Hartmann (2000) and International Monetary Fund (2011, Ch. 3)

audience with our own preconceptions and opinions. Therefore, we have largely refrained from any editorial commentary regarding the advantages and disadvantages of the measures contained in this survey, and our inclusion of a particular approach should not be construed as an endorsement or recommendation, just as omissions should not be interpreted conversely. We prefer to let the users, and experience, be the ultimate judges of which measures are most useful.

Our motivation for providing open-source software for these measures is similar: we wish to encourage more research and development in this area by researchers from all agencies, disciplines, and industries. Having access to working code for each measure should lower the entry cost to the field. We have witnessed the enormous leverage that the "wisdom of crowds" can provide to even the most daunting intellectual challenges—for example, the Netflix Prize, the DARPA Network Challenge, and Amazon's Mechanical Turk—and hope that this survey may spark the same kind of interest, excitement, and broad engagement in the field of systemic risk analytics. Accordingly, this survey is intended to be a living document, and we hope that users will not only benefit from these efforts, but will also contribute new analytics, corrections and revisions of existing analytics, and help expand our understanding of financial stability and its converse. In the long term, we hope this survey will evolve into a comprehensive library of systemic risk research, a knowledge base that includes structured descriptions of each measurement methodology, identification of the necessary data inputs, source code, and formal taxonomies for keyword tagging to facilitate efficient online indexing, searching, and filtering.

Although the individual models and methods we review were not created with any classification scheme in mind, nonetheless, certain commonalities across these analytics allow us to cluster the techniques into clearly defined categories, e.g., based on the types of inputs required, analysis performed, and outputs produced. Therefore, we devote a significant amount of attention in this paper to organizing systemic risk analytics into several taxonomies that will allow specific audiences such as policymakers, data and information-technology staff, and researchers to quickly identify those analytics that are most relevant to their unique concerns and interests.

However, the classifications we propose in this paper are necessarily approximate. Each risk measure should be judged on its own merits, including the data required and available, the sensitivities of the model, and its general suitability for capturing a particular aspect

of financial stability. Because our goal for each taxonomy is to assist users in their search for a particular risk measure, creating a single all-inclusive classification scheme is neither possible nor desirable. A number of papers we survey are internally diverse, defying unique categorization. Moreover, the boundaries of the discipline are fuzzy in many places and expanding everywhere. An organizational scheme that is adequate today is sure to become obsolete tomorrow. Not only will new approaches emerge over time, but innovative ideas will reveal blind spots and inadequacies in the current schemas, hence our taxonomies must also evolve over time.

For our current purposes, the most important perspective is that of policymakers and regulators since they are the ones using systemic risk models day-to-day. Therefore, we begin in Section 2 with a discussion of systemic risk analytics from the supervisory perspective, in which we review the financial trends that motivate the need for greater disclosure by systemically important financial institutions, how regulators might make use of the data and analytics produced by the OFR, and propose a different taxonomy focused on supervisory scope. In Section 3, we turn to the research perspective and describe a broader analytical framework in which to compare and contrast various systemic risk measures. This framework naturally suggests a different taxonomy, one organized around methodology. We also include a discussion of non-stationarity, which is particularly relevant for the rapidly changing financial industry. While there are no easy fixes to time-varying and state-dependent risk parameters, awareness is perhaps the first line of defense against this problem. For completeness, we also provide a discussion of various data issues in Section 4, which includes a summary of all the data required by the systemic risk analytics covered in this survey, a review of the OFR's ongoing effort to standardize legal entity identifers, and a discussion of the trade-offs between transparency and privacy and how recent advances in computer science may allow us to achieve both simultaneously. We conclude in Section 5.

2   Supervisory Perspective

The Financial Crisis of 2007–2009 was a deeply painful episode to millions of people; hence, there is significant interest in reducing the likelihood of similar events in the future. The Dodd Frank Act clearly acknowledges the need for fuller disclosure by systemically important financial institutions (SIFIs), and has endowed the OFR with the statutory authority to compel such entities to provide the necessary information (including subpeona power). Nev-

ertheless, it may be worthwhile to consider the changes that have occurred in our financial system which justify significant new disclosure requirements and macroprudential supervisory practices. A number of interrelated long-term trends in the financial services industry suggest that there is more to the story than a capricious, one-off "black-swan" event that will not recur for decades. These trends include the gradual deregulation of markets and institutions, disintermediation away from traditional depositories, and the ongoing phenomenon of financial innovation.

2.1 Trends in the Financial System

Innovation is endemic to financial markets, in large part because competition tends to drive down profit margins on established products. A significant aspect of recent innovation has been the broad-based movement of financial activity into new domains, exemplified by the growth in mortgage securitization and "shadow banking" activities. For example, Gorton and Metrick (2010) document the strong growth since the 1980s in repo and money-fund assets, and Loutskina and Strahan (2009) demonstrate that the widespread availability of securitization channels has improved liquidity in mortgage markets, reducing the sensitivity of credit supply to the idiosyncratic financial conditions of individual banks. Facilitating these institutional changes are underlying advances in modeling portfolio credit risk, legal and technical developments to support electronic mortgage registration, and the expansion of markets for credit derivatives. Another factor is the burden of supervision and regulation, which falls more heavily on established institution types such as traditional banks and broker-dealers, and relatively lightly on hedge funds and private equity firms.

As innovation and alternative investments become more significant, the complexity of the financial system grows in tandem—and size matters. In many cases, financial innovation has effectively coincided with deregulation, as new activities have tended to expand most among less regulated, non-traditional institutions. For example, in the 1980s, the hedge-fund industry was well established but small enough that its activities had little effect on the rest of the system. By the late 1990s, hedge-fund assets and activities had become so intertwined with global fixed-income markets that the demise of a single hedge fund—Long Term Capital Management (LTCM)—was deemed potentially so disruptive to financial stability that the Federal Reserve Bank of New York felt compelled to broker a bailout. Securitization is particularly important in this context: it effectively disintermediates and deregulates

7

simultaneously by moving assets off the balance sheets of highly regulated, traditional depositories, and into less regulated special purpose vehicles. Adrian and Shin (2009) connect the growth in shadow banking to securitization, arguing that the latter has enabled increases in leverage by reducing idiosyncratic credit risk at originating institutions. As securitization activity expanded, the balance sheets of securities firms such as Lehman Brothers ballooned, potentially increasing the fragility of the system as a whole. Adrian and Shin (2009) demonstrate the procyclicality of this (de-)leveraging effect through the recent boom and crisis. The collapse in the asset-backed securitization market that followed the crisis was, in effect, a re-intermediation, and re-regulation has emerged in the form of the Dodd Frank Act in the U.S. and similar legislation in the United Kingdom and the European Union. Even innovation has taken a holiday, with structured products falling out of favor and investors moving closer to cash and its equivalents.

Over the longer term, however, broader trends have also involved disintermediation. Feldman and Lueck (2007) update an earlier study of long-term structural trends in financial markets by Boyd and Gertler (1994), and using adjusted flow-of-funds data, they show that banks have employed a variety of techniques, including securitization, to recover market share lost in the 1980s and 1990s. However, their statistics also show dramatic growth in market share for "other financial intermediaries", which increases from less than 10% in 1980 to roughly 45% in 2005 (see Feldman and Lueck (2007, Figure 3)). Even this is a gross underestimate because "other financial intermediaries" does not include the hedge fund industry. Accompanying this broader trend of disintermediation is the secular growth in the finance and insurance industries as a share of the U.S. and global economies. There is considerable anecdotal evidence for this growth in recent years—in numbers, assets, employees, and diversity—and more objective measures such as per capita value-added and salary levels confirm this informal impression. Total employment of the finance and insurance sectors has continued to rise, even in recent decades as the spread of automation has eliminated many back-office jobs. This pattern is part of a larger trend in the U.S. economy where, according to nominal U.S. GDP data from 1947 to 2009, service industries have become an increasingly larger proportion of the U.S. economy than goods-producing industries since the post-war period. The finance and insurance have grown almost monotonically during that period, in contrast to many other goods-producing sectors such as manufacturing. One implication of these trends is that the repercussions of sector-wide shocks to the financial system are likely

to be larger now than in the past.

Closely related to the growth of the financial sector is the intensity of activity in that sector. This is partly the result of innovations in telecommunications and computer technology, and partly due to financial innovations that encourage rapid portfolio rebalancing, such as dynamic hedging, portfolio insurance, and tracking indexes.[5] Whether measured by trading volume, number of transactions, the total assets deployed, or the speed with which transactions are consummated, the pace of financial activity has increased dramatically, even over the last decade. Improvements in computation, connectivity, trading, social and financial networking, and globalization have facilitated ever faster and more complex portfolio strategies and investment policies. The co-location of high-frequency trading algorithms at securities exchanges is perhaps the most extreme example, but the "paperwork crisis" of the late 1960s was an early indication of this trend. The implication for regulatory supervision is that the relatively leisurely pace of quarterly financial reporting and annual examinations is becoming increasingly inadequate. Moreover, legacy supervisory accounting systems sometimes fail to convey adequately the risk exposures from new complex contingent contracts, and from lightly regulated markets with little or no reporting requirements. In fact, supervisors do not even have consistent and regularly updated data on some of the most basic facts about the industry, such as the relative sizes of all significant market segments.

A related concern is whether the systemic consequences of shocks to these sectors might be more or less severe than among the more traditional institutional segments. This is largely an open question because so little is known about systemic exposures in the shadow banking sector. Feldman and Lueck (2007, pp. 48–49) conclude with a plea for more detailed information, since "good policy on banking requires a solid sense of banks' market share." In a world of interconnected and leveraged institutions, shocks can propagate rapidly throughout the financial network, creating a self-reinforcing dynamic of forced liquidations and downward pressure on prices.

Lack of transparency also hampers the ability of firms to protect themselves. Market

---

[5]Even the simplest measure, such as the average daily trading volume in the S&P 500 index exhibits an increase of three orders of magnitude over the last half century, from 3 million shares in 1960 to just over 4 billion shares as of September 1, 2011. The growth in equity market trading is only a lower bound for the growth in total financial-market activity. It does not include the explosive growth in the many exchange-traded and over-the-counter derivatives since the 1970s, including the introduction of S&P 500 index futures contracts. It also ignores the broad expansion of securitization techniques, which have converted large swaths of previously illiquid loan contracts into bonds that trade actively in secondary markets.

participants may know their own counterparties, but no individual firm can peer more deeply into the counterparty network to see all of the interconnections through which it can be affected. Two familiar examples illustrate this more general problem. Participants who had purchased CDS protection from AIG Financial Products were unknowingly exposed to wrong-way risk because they could not see the full extent of AIG's guarantee exposures to others, and Lehman Brothers disguised the full extent of its leverage from other participants via its "Repo 105" transactions. Because trading firms must maintain secrecy around their portfolio exposures to remain profitable, the opaqueness of the financial network will never resolve itself solely through market mechanisms.

2.2 Policy Applications

Having made the case for additional disclosure by SIFIs, a natural response by industry stakeholders is to ask how such disclosure and systemic risk analytics be used and why the financial industry should be a willing participant? While the details of macroprudential and systemic risk policy decisions are beyond the scope of this paper, a few general observations about uses and abuses may be appropriate. Alexander (2010) provides a useful perspective on this issue in his outline of four distinct policy applications of systemic risk measures: (a) by identifying individual institutions posing outsized threats to financial stability (i.e., SIFIs), metrics can help in targeting heightened supervisory standards; (b) by identifying specific structural aspects of the financial system that are particularly vulnerable, metrics can help policymakers identify where regulations need to be changed; (c) by identifying potential shocks to the financial system posing outsized threats to stability (e.g., asset price misalignments), metrics may help guide policy to address those threats; and (d) by indicating that the potential for financial instability is rising (i.e., providing early warning signals), metrics can signal to policymakers a need to tighten so-called macroprudential policies.

The benefits of systemic risk measures in *ex post* forensic analysis of market performance and behavior in the wake of systemic events should not be underestimated. Such analyses are routinely performed in other industries such as transportation, and may help identify institutional weaknesses, regulatory lapses, and other shortcomings that lead to much-needed reforms.[6] In fact, apart from occasional Inspector General's reports and presidential commis-

---

[6]See Fielding, Lo, and Yang (2011) for a detailed description of how the National Transportation Safety Board has played a critical role in improving safety in the transportation industry despite having no regulatory responsibility or authority.

sions, we have not institutionalized regular and direct feedback loops between policymaking and their outcomes in the financial sector. The ability to identify underperforming policies and unintended consequences quickly and definitively is one of the most effective ways of improving regulation, and measurement is the starting point.

With respect to early warning indicators of impending threats to financial stability, three important caveats apply. First, reliable forecast power alone will not solve the supervisory decision problem because there is no single "pressure gauge" that captures the full state of an intricate, multifaceted financial system. There will always be noise and conflicting signals, particularly during periods of financial distress. Moreover, since many of the metrics described here can be used with different time periods, firms, countries, asset classes, market sectors, and portfolios, the "curse of dimensionality" applies. In a real decision environment, techniques will be needed for sifting through such conflicting signals.

Second, there is the problem of statistical regime shifts, which are particularly relevant for systemic events. Adding model structure can improve conditional forecasts, especially in a shifting environment, but even if we know the correct structural model—a heroic assumption, particularly *ex ante*—obtaining a reliable statistical fit is a nontrivial matter. Of course, in practice, we can never be sure about the underlying structure generating the data. For example, in the run-up to the recent crisis, knowledgeable and credible experts were found on both sides of the debate surrounding the over- or under-valuation of U.S. residential real estate.

Third, to the extent that the Lucas critique applies (see Section 2.3), early warning indicators may become less effective if individuals change their behavior in response to such signals. Apart from the question of whether or not such indicators are meant for regulators' eyes only or for the public, this possibility implies an ongoing need to evaluate the efficacy of existing risk analytics and to develop new analytics as old measures become obsolete and new systemic threats emerge. This is one of the primary reasons for the establishment of the OFR.

As to why the financial industry should willingly participate in the OFR's research agenda, perhaps the most obvious and compelling reason is that all financial institutions benefit from financial stability, and most institutions are hurt by its absence. For example, the breakdown in stability and liquidity, and the collapse of asset prices in the fall and winter of 2008–2009 were an enormous negative-sum event that imposed losses on most participants.

In the aftermath of this crisis, there is near unanimity that firm-level risk management and supervision have limitations, and that the fallacy of composition applies: patterns exist in market dynamics at the system level that are distinct from the simple aggregation of the behavior of the individual participants.[7]

Moreover, while all firms share the benefits of financial stability, market mechanisms do not exist to force firms to internalize the full cost of threats to stability created by their own activities. To address these externalities, systemic risk measures may be used to provide more objective and equitable methods for calibrating a Pigouvian tax on individual SIFIs, as proposed by Acharya and Richardson (2009), or the Basel Committee's (2011) capital surcharge on global systemically important banks (G-SIBs). These proposals are controversial. The Clearing House—a trade association of 17 of the world's largest commercial banks responded that, "there are significant open questions regarding the purported theoretical and policy foundations, as well as the appropriate calibration, for a G-SIB surcharge". As with any policy intervention, we should always be prepared to address the possibility of unintended consequences.

Another reason firms are not always penalized for their risky behavior is the existence of a safety net, created by government policy either explicitly (e.g., deposit insurance) or implicitly (e.g., too-big-to-fail policies). It has long been recognized that both deposit insurance and the discount window can encourage banks to take risks that might endanger their solvency.[8] In hindsight, it is clear that, throughout the recent crisis, both regulators and market participants failed to act in a timely fashion to curtail excessive leverage and credit expansion.

It is tempting to attribute such supervisory forbearance to some form of regulatory capture.[9] However, forbearance might also be motivated by indecisiveness, which can be ex-

---

[7]See Danielsson and Shin (2003) for an evocative example of the fallacy of composition. This basic principle is reflected in many of the measures here.

[8]Acharya and Richardson (2009) discuss the general role of government mispricing of risk in encouraging risky behavior, and the papers in Lucas (2010) propose better pricing models for government guarantees. For a recent analysis of the moral hazard inherent in deposit insurance, see Demirguc-Kunt, Kane, and Laeven (2008). On the historical understanding of the moral hazard issues at the time of the FDIC's creation, see Flood (1992). Regarding the moral hazard inherent in the lender of last resort function, see Rochet and Vives (2004). For an analysis of the historical understanding, see Bordo (1990) or Bignon, Flandreau, and Ugolini (2009).

[9]There is an extensive literature on forbearance and regulatory capture, well beyond the scope of this paper. For examples dating from the aftermath of the 1980s S&L crisis, see Kane (1989) and Boot and Thakor (1993). Two recent studies consider these arguments in the context of the recent crisis: Huizinga and Laeven (2010) and Brown and Din (2011).

acerbated by limited information and penalties regulators may face for making mistakes. Regulatory action in the face of unsafe or unsound practices typically involves formal interruptions of ongoing business activities, e.g., via cease-and-desist orders or the closure of an institution. Such decisions are not lightly made because they are fraught with uncertainty and the stakes are high. Waiting for unequivocal evidence of trouble can allow losses to accumulate, especially if the state of the institution is observed infrequently and measured with error, and managers and regulators are gambling on a significant reversal (Benston and Kaufman, 1997).

In fact, the loss function for supervisory mistakes is highly asymmetric between Type-I (premature closure) and Type-II (forbearance) errors. Regulators expect to be punished, e.g., reprimanded or sued, for acting too soon by closing a solvent firm. The opposite mistake—waiting until after a firm defaults on its obligations—puts the regulator in the role of cleaning up a mess created by others, but the perceived penalty is much smaller. At any point in time, this asymmetry creates strong incentives for supervisors to wait one more day, either for the arrival of unequivocal information to support a particular choice, or for the decision to become moot through the failure of the institution.[10] In these circumstances, improved techniques for measuring threats can significantly reduce the likelihood of policy mistakes.

While economic incentives alone can create a bias toward forbearance, these tendencies are likely to be exacerbated by well-known behavioral tendencies. "Prompt corrective action" can avert large accumulated losses, but such prophylactic responses always introduce the possibility of errors in supervisory decisions, with negative short- and long-term consequences to the regulator. Hardwired behavioral responses to "double down" and become more risk-tolerant when faced with sure losses only make matters worse in these situations.[11]

More generally, accurate systemic risk metrics can foster better *ex post* accountability for regulators: if they knew, or should have known, of systemic dangers *ex ante*, but failed

---

[10]In the words of Shakespeare's Hamlet (Act III, Scene 1), "Thus conscience does make cowards of us all." Boot and Thakor (1993) present a similar argument in the context of a detailed model, in which regulators act to preserve their valued reputations, which would be damaged by the revelation of a premature closure. The result is a pooling equilibrium in which the asymmetric reputational costs of a premature closure vs. forbearance lead all regulators to mimic each other's closure policies. However, their model allows no possibility for regulators to improve their monitoring technology. Incentives are also supported in the model by a second period after the close/wait decision that allows bankers to "gamble for resurrection".

[11]See Kahneman and Tversky (1979) for the loss aversion phenomenon, and Lo (2011, Section 5) for a discussion of its relevance for risk managers, policymakers, and rogue traders.

to act, systemic risk metrics can provide the basis for remedial action. However, once again, there may be an unintended consequence in that silence from an informed regulator might be construed as tacit consent. Therefore, systemic risk monitoring must be structured so as not to absolve market participants of responsibility for managing their own risks.

2.3   The Lucas Critique and Systemic Risk Supervision

No policy discussion would be complete without addressing the potential impact of feedback effects on human behavior and expectations, i.e., the Lucas (1976, p. 41) critique, that "any change in policy will systematically alter the structure of econometric models". Of course, we have little to add to the enormous literature in macroeconomics on this topic, and refer readers instead to the excellent recent essay by Kocherlakota (2010) in which he reviews this important idea and its influence on modern macroeconomics and monetary policy.

As a starting point, we presume that the Lucas critique applies to systemic risk supervision. Measurement inevitably plays a central role in regulatory oversight and in influencing expectations. Imagine conducting monetary policy without some measure of inflation, GDP growth, and the natural rate of unemployment. Given that systemic risk monitoring will provoke institutional and behavioral reactions, the relevant questions revolve around the nature and magnitude of the impact. The first observation to be made about the Lucas critique is that it has little bearing on the importance of accurate metrics for systemic risk. By yielding more accurate inputs to policy decisions, these measures should have important first-order benefits for systemic stability, regardless of whether and how fully individual and institutional expectations might discount the impact of such policies.

The second observation regarding the Lucas critique is related to the fact that many of the analytics contained in this survey are partial-equilibrium measures. Therefore, by definition they are subject to the Lucas critique to the extent that they do not incorporate general-equilibrium effects arising from their becoming more widely used by policymakers. The same can be said for enterprise-wide risk management measures—once portfolio managers and chief risk officers are aware of the risks in their portfolios and organizations, they may revise their investment policies, changing the overall level of risk in the financial system. This may not be an undesirable outcome. After all, one of the main purposes of early warning signals is to encourage individuals to take action themselves instead of relying solely on government intervention. However, this thought experiment does not necessarily

correspond to a dynamic general equilibrium process, but may involve a "phase transition" from one equilibrium to another, where the disequilibrium dynamics takes weeks, months, or years, depending on the frictions in the system. The Lucas critique implies that the general-equilibrium implications of systemic risk policies must be studied, which is hardly controversial. Nevertheless, partial-equilibrium measures may still serve a useful purpose in addressing short-term dynamics, especially in the presence of market imperfections such as transactions costs, non-traded assets, incomplete markets, asymmetric information, externalities, and limited human cognitive abilities.

Finally, rational expectations is a powerful idea for deducing the economic implications of market dynamics in the limiting case of agents with infinite and instantaneous cognitive resources. However, recent research in the cognitive neurosciences and in the emerging field of neuroeconomics suggest that this limiting case is contradicted by empirical, experimental, and evolutionary evidence. This is not particularly surprising in and of itself, but the more informative insights of this literature have to do with the specific neural mechanisms that are involved in expectations, rational and otherwise.[12] This literature implies that rational expectations may only be one of many possible modes of economic interactions between *Homo sapiens*, and the failure of dynamic stochastic general equilibrium models to identify the recent financial crisis seems to support this conclusion.

For these reasons, we believe the Lucas critique does not vitiate the need for measures of systemic risk; on the contrary, it buttresses the decision to create the OFR as a research-centric institution. We are still in the earliest days of understanding the elusive and multi-faceted concept of systemic risk, and the fact that markets and individuals adapt and evolve in response to systemic measurement only reinforces the need for ongoing research.

2.4 Supervisory Taxonomy

A second taxonomy for the analytics reviewed in this survey is along the lines of supervisory scope, which is of particular interest to policymakers. Institutionally, individual regulators' responsibilities and activities are typically segregated by industry subsector. The jurisdictional boundaries that separate the regulatory purview of the individual agencies provide clarity for regulated entities, and allow supervisors to develop focused expertise in particular

---

[12]For example, Lo (2011) provides a review of the most relevant research in the cognitive neurosciences for financial crises, in which recent studies have shown that the regions of the brain responsible for mathematical reasoning and logical deduction are forced to shut down in the face of strong emotional stimuli.

areas of the financial system. A given systemic risk metric may be more or less relevant for a particular regulator depending on the regulator's supervisory jurisdiction. Because it is likely that a given crisis will be triggered by events at a specific institution with a clearly identified primary regulator, e.g., LTCM or Lehman, having metrics that are tailored to specific institutional types and business models may help pinpoint dangers in those institutions and sound the alarm for the relevant regulator. For example, measures of equity market liquidity will likely interest the securities market supervisors more than housing regulators.

However, by definition, threats to financial stability involve many institutions simultaneously and typically affect the system as a whole. Among others, Brunnermeier, Crockett, Goodhart, Persaud, and Shin (2009, pp. 6–10) emphasize the distinction between microprudential regulation (especially the capital-focused Basel system), and macroprudential regulation. The former is focused on prudential controls at the firm level, while the latter considers the system as a whole.[13] Although the impact of systemic events is a macroprudential concern, particular metrics of threats to financial stability may by applicable at either a microprudential or a macroprudential level (or sometimes both).

To this end, grouping systemic risk analytics by supervisory scope will yield two broad categories, microprudential and macroprudential analytics, and within the former category, we can further categorize them by institutional focus: securities and commodities, banking and housing, insurance and pensions, and general applications. This new taxonomy is summarized in Table 2, and we describe each of these categories in more detail below.

2.4.1 Microprudential Measures: Securities and Commodities

Securities and commodities market regulators have jurisdiction over a broad range of secondary market and inter-institution trading. For example, the U.S. Securities and Exchange Commission (SEC) and Commodities Futures Trading Commission (CFTC) together regulate a range of markets, including equities, commodities, and currencies, along with the securities firms active in those markets such as investment managers, mutual funds, broker/dealers, and, post-Dodd Frank, hedge funds. Similar supervisors exist in other countries, although the details of regulatory authority naturally vary across geopolitical boundaries. Several of the measures of fragility surveyed here focus on this market segment. Pojarliev and Levich (2011) look for patterns of coordinated behavior, i.e., "crowded trades", in high-

---

[13]See also Hanson, Kashyap, and Stein (2011), and Bank of England (2009).

Systemic Risk Measure	Section
Microprudential Measures—Securities and Commodities:	
Crowded Trades in Currency Funds	F.4
Equity Market Illiquidity	F.5
Serial Correlation and Illiquidity in Hedge Fund Returns	F.6
Broader Hedge-Fund-Based Systemic Risk Measures	F.7
Microprudential Measures—Banking and Housing:	
Network Analysis and Systemic Financial Linkages	B.2
Simulating a Credit Scenario	B.3
Simulating a Credit-and-Funding-Shock Scenario	B.4
Bank Funding Risk and Shock Transmission	B.6
The Option iPoD	C.3
Multivariate Density Estimators	C.4
Simulating the Housing Sector	C.5
Consumer Credit	C.6
Lessons from the SCAP	D.2
A 10-by-10-by-10 Approach	D.3
Distressed Insurance Premium	E.2
Microprudential Measures—Insurance and Pensions:	
Granger-Causality Networks	B.5
Mark-to-Market Accounting and Liquidity Pricing	B.7
Microprudential Measures—General Applications:	
The Default Intensity Model	B.1
Contingent Claims Analysis	C.1
Mahalanobis Distance	C.2
CoVaR	E.1
Co-Risk	E.3
Marginal and Systemic Expected Shortfall	E.4
Risk Topography	F.1
The Leverage Cycle	F.2
Macroprudential Measures:	
Costly Asset-Price Boom/Bust Cycles	A.1
Property-Price, Equity-Price, and Credit-Gap Indicators	A.2
Macroprudential Regulation	A.3
Principal Components Analysis	C.7
GDP Stress Tests	D.1
Noise as Information for Illiquidity	F.3

Table 2: Taxonomy of systemic risk measures by supervisory scope.

frequency trading data for currency funds. Khandani and Lo (2011) consider two distinct measures of liquidity in equity markets. Getmansky, Lo, and Makarov (2004) and Chan, Getmansky, Haas, and Lo (2006b, 2006b) also focus on liquidity, in this case for hedge funds, where serial correlation in reported returns can appear as an artifact of reporting conventions in illiquid markets.

2.4.2 Microprudential Measures: Banking and Housing

Depository institutions form the core constituency for the cluster of banking regulators, including central banks, deposit insurers, and bank chartering agencies. Residential mortgage originators, such as thrifts, building and loan societies, and mortgage banks also fall into this grouping, along with housing GSEs such as Fannie Mae, Freddie Mac, and the Federal Home Loan (FHL) banks in the U.S. Within this class, Fender and McGuire (2010a) look for binding funding constraints in aggregate balance sheet data for international banking groups. Merton and Bodie (1993) focus on the corporate financing, especially leverage, of insured depositories. Khandani, Kim, and Lo (2010) consider aggregate patterns in consumer lending via credit-risk forecasts estimated from detailed credit-card data. Huang, Zhou, and Zhu (2009a) calculate a hypothetical insurance premium based on firms' equity prices and CDS spreads; they apply this to a sample of banks. Khandani, Lo, and Merton (2009) examine coordinated increases in homeowner leverage, due to a one-way "ratchet" effect in refinancing behavior. Capuano (2008) and Segoviano and Goodhart (2009) use techniques from information theory to extract implied probabilities of default (iPoD) from equity and equity option prices, applying this technique to samples of commercial and investment banks. Chan-Lau, Espinosa, and Sole (2009) and Duffie (2011) construct financial network models, and take banking firms as the primary sample of interest.

2.4.3 Microprudential Measures: Insurance and Pensions

Pension and insurance regulators, such as the European Insurance and Occupational Pensions Authority (EIOPA) in Europe and the Pension Benefit Guaranty Corporation (PBGC) and state insurance departments in the U.S., are the focus of the third microprudential category in our taxonomy. Relatively few of the studies in our sample deal directly with pension funds or insurance companies, despite the fact that the recent crisis actively involved these institutions. An exception is Billio, Getmansky, Lo, and Pelizzon (2010), who include in-

surance as one of four industry sectors in a latent factor model used to identify patterns of risk concentration and causation. An insurance company subsidiary, AIG Financial Products, played a prominent role in the recent crisis as a seller of credit protection on subprime mortgage securitizations, and pension funds were among the buyers of the same.[14] The lack of easily accessible data in these industries is a significant factor: pension-fund and insurance-company portfolio holdings are not widely available, unlike equity and bond market benchmark indexes that would broadly track their performance. Sapra (2008) considers issues arising from historical and mark-to-market accounting for both insurance companies and banks.

2.4.4 Microprudential Measures: General Applications

On the other hand, accounting and market price data for large financial firms are widely available, and a number of fragility measures based on stock-market data could be applied to any or all of the microprudential categories just listed. Like Merton and Bodie (1993), Geanakoplos (2010) similarly focuses on institutional leverage, but he envisions a much broader scope of applicability than just banks. Gray and Jobst (2010) use CDS spreads in a contingent claims analysis of financial firm risk. Adrian and Brunnermeier's (2010) conditional value at risk (CoVaR) and the International Monetary Fund's (2009b) related "Co-Risk" models of shared exposures similarly rely on firm-level market prices.[15] The Mahalonobis distance metric of Kritzman and Li (2010) is a statistical model that could, in principle, be applied to any time series.

2.4.5 Macroprudential Measures

Although the boundaries that support efficient institutional specialization among regulators serve many practical purposes, nevertheless they sometimes create the jurisdictional gaps within which risky activities are most likely to go undetected. These gaps are covered by

---

[14]AIG Financial Products (AIGFP) is an example of a firm that does not fit neatly into the microprudential regulatory framework. Although it was an insurance company subsidiary, it was supervised by a domestic housing regulator, the Office of Thrift Supervision (OTS), without deep expertise in the credit derivatives that were AIGFP's specialty. Moreover, AIGFP was headquartered in London, adding a geographic obstacle. Ashcraft and Schuermann (2008) describe subprime securitizations with the example of a pension fund investor.

[15]The default intensity model of Giesecke and Kim (2009), the distressed insurance premium (DIP) of Huang, Zhou, and Zhu (2009a), and the systemic expected shortfall (SES) of Acharya, Pedersen, Philippon, and Richardson (2010) also satisfy this general description.

macroprudential regulation, which is, of course, not new.[16] Two of the oldest elements of the U.S. regulatory safety net are motivated by macroprudential concerns. The discount window, which provides emergency liquidity support to "innocent bystander" banks in a systemic crisis, was created with the founding of the Federal Reserve in 1913. Deposit insurance— created at the federal level in 1933 with the Federal Deposit Insurance Corporation (FDIC)— discourages bank runs and provides for orderly resolution of failing depositories.

However, it has been almost eighty years since the creation of the FDIC, and nearly a century since the founding of the Fed, and the intervening decades have witnessed a steady disintermediation away from traditional depository institutions. Recent decades have shown strong growth in direct capital-market access by large borrowers, derivatives markets, managed investment portfolios (including mutual funds, ETFs, and hedge funds), and various forms of collateralized borrowing (including asset-backed and mortgage-backed securitization and repurchase agreements). As a result, when the crisis struck in force in the Fall of 2008, large segments of the financial system did not have immediate access to orderly resolution (FDIC) or lender-of-last-resort (Fed) facilities.

Macro-level metrics tend to concentrate on aggregate imbalances. As a result, they are frequently intended to serve as early-warning signals, tracking the buildup of unsustainable tensions in the system. For the same reason, there is also a tendency to use macroeconomic time series and official statistics in these measures. For example, Borio and Drehmann (2009b) look for simultaneous imbalances in broad indicators of equity, property, and credit markets. Alfaro and Drehmann (2009) examine the time series of GDP for signs of weakening in advance of a crisis. Hu, Pan, and Wang (2010) extract an indicator of market illiquidity from the noise in Treasury prices. The absorption ratio of Kritzman, Li, Page, and Rigobon (2010) measures the tendency of markets to move in unison, suggesting tight coupling. Alessi and Detken (2009) track anomalous levels in macroeconomic time series as possible indicators of boom/bust cycles.

---

[16]Clement (2010) traces the usage of the term "macroprudential" back to the 1970s, citing (p. 61) in particular a Bank of England background paper from 1979, "This 'macroprudential' approach considers problems that bear upon the market as a whole as distinct from an individual bank, and which may not be obvious at the micro-prudential level." Etymology aside, macroprudential supervision has a longer history.

2.5 Event/Decision Horizon Taxonomy

Decision-making is a critical activity for policymakers, who must choose whether, when, and how to intervene in the markets. In this context, the informativeness of a systemic risk metric over time—especially relative to a decision horizon or the onset of a systemic event—is significant. Accordingly, we can classify risk analytics into three temporal categories: pre-event, contemporaneous, and post-event analytics. There is obvious benefit from measures that provide early warning of growing imbalances or impending dangers; forewarned is often forearmed. However, even strictly contemporaneous signals of market turmoil can be useful in allocating staff and other supervisory infrastructure during an emerging crisis; reaction time matters, particularly as events are unfolding. And there is also a role for ex-post analysis in maintaining accountability for regulators (see the discussion in Section 2.2 and Borio (2010)) and generating forensic reports of systemic events. This event- and decision-horizon classification scheme is summarized in Table 3.

2.5.1 Ex Ante Measures: Early Warning

In an ideal world, systemic monitoring would work like the National Weather Service, providing sufficiently advance notice of hurricanes for authorities and participants to intervene by pre-positioning staff and resources, minimizing exposures, and planning for the impending event and immediate aftermath. This may be too much to hope for in the case of financial stability. Systemic shocks can arrive from many directions, such as the sovereign default that triggered the LTCM crisis, the algorithmic feedback loop of the May 6, 2010 "Flash Crash", or the speculative attacks that have repeatedly plagued small-country financial systems. Moreover, unlike hurricanes, many significant threats involve active subterfuge and evasive behavior. For example, institutions vulnerable to contagious runs, like Lehman Brothers in the run-up to its 2008 collapse, have strong incentives to avoid revealing information that could trigger a self-reinforcing attack.[17] Therefore, tracking a multitude of threats will require a diversity of monitoring techniques.

We define "early warning" models as measures aspiring to a significant degree of forecast power. Several of the macroprudential measures mentioned above are intended to identify

---

[17]Per the bankruptcy court report, Valukas (2010, p. 732),"Lehman employed off-balance sheet devices, known within Lehman as 'Repo 105' and 'Repo 108' transactions, to temporarily remove securities inventory from its balance sheet, usually for a period of seven to ten days, and to create a materially misleading picture of the firm's financial condition in late 2007 and 2008."

Systemic Risk Measure	Section
Ex Ante Measures—Early Warning:	
Costly Asset-Price Boom/Bust Cycles	A.1
Property-Price, Equity-Price, and Credit-Gap Indicators	A.2
The Default Intensity Model	B.1
Network Analysis and Systemic Financial Linkages	B.2
Simulating the Housing Sector	C.5
Consumer Credit	C.6
GDP Stress Tests	D.1
Distressed Insurance Premium	E.2
The Leverage Cycle	F.2
Serial Correlation and Illiquidity in Hedge Fund Returns	F.6
Broader Hedge-Fund-Based Systemic Risk Measures	F.7
Ex Ante Measures—Counterfactual Simulation and Stress Tests:	
Simulating a Credit Scenario	B.3
Simulating a Credit-and-Funding-Shock Scenario	B.4
Lessons from the SCAP	D.2
A 10-by-10-by-10 Approach	D.3
Marginal and Systemic Expected Shortfall	E.4
Contemporaneous Measures—Fragility:	
Granger-Causality Networks	B.5
Contingent Claims Analysis	C.1
The Option iPoD	C.3
Multivariate Density Estimators	C.4
CoVaR	E.1
Co-Risk	E.3
Contemporaneous Measures—Crisis Monitoring:	
Bank Funding Risk and Shock Transmission	B.6
Mahalanobis Distance	C.2
Principal Components Analysis	C.7
Noise as Information for Illiquidity	F.3
Crowded Trades in Currency Funds	F.4
Equity Market Illiquidity	F.5
Ex Post Measures—Forensic Analysis:	
Macroprudential Regulation	A.3
Mark-to-Market Accounting and Liquidity Pricing	B.7
Ex Post Measures—Orderly Resolution:	
Risk Topography	F.1

Table 3: Taxonomy of systemic risk measures by event/decision time horizon.

accumulating imbalances, and thereby to have some forecast power for systemic events while using an observation or update interval longer than daily or weekly. These include Borio and Drehmann (2009b) and Alessi and Detken (2009), who use quarterly data, and Alfaro and Drehmann (2009), whose model is updated only annually. Higher-frequency measures with some potential forecast power include Khandani, Kim, and Lo's (2010) model of consumer credit risk, the default intensity model of Giesecke and Kim (2009), Huang, Zhou, and Zhu's (2009a) DIP metric, the hedge fund measures of Chan, Getmansky, Haas, and Lo (2006b, 2006b), the mortgage ratcheting model of Khandani, Lo, and Merton (2009), the cross-funding network analysis of Chan-Lau, Espinosa, and Sole (2009), and Getmansky, Lo, and Makarov's (2004) model of serial correlation and illiquidity in hedge fund returns.

2.5.2 Ex Ante Measures: Counterfactual Simulation and Stress Tests

Predictive models assign probabilities to possible future events, conditional on current and past observations of the system. Another prospective approach to assessing the vulnerability of a system is to examine its behavior under counterfactual conditions. Stress testing is codified in regulation and international standards, including the Basel accord. It is applied, for example, in the Federal Reserve's (2009) SCAP study. As a matter both of regulatory policy and traditional risk management, the process can be viewed as a means to identify vulnerabilities in the portfolio—i.e., combinations of external factor outcomes causing unacceptably large losses—and ways to defend against those influences. A related approach is reverse stress testing, in which a portfolio outcome (typically insolvency) is fixed, and a search is undertaken for scenarios that could provoke this level of distress. A stress test typically draws its scenarios either from actual historical stress episodes or hypothesizes them via expert opinion or other techniques. Breuer, Jandačka, Rheinberger, and Summer (2009), for example, emphasize three characteristics of well designed stress scenarios—plausibility, severity, and suggestiveness of risk-reducing action—and present an algorithm for searching within a "plausible" subset of the space of external factor outcomes for the scenario that generates the largest portfolio loss. Simultaneously targeting both severity and plausibility introduces a natural tension, since outlandish scenarios are likely to have painful ramifications. As a policy matter, if the goal of the exercise is simply to explore portfolio sensitivities (i.e., not to calibrate required capital or other regulatory constraints), then this trade-off is less immediate.

Stress scenarios are frequently stated in terms of possible values for macroeconomic fundamentals. A straightforward example is Alfaro and Drehmann (2009), who consider the behavior of GDP around 43 post-1974 crises identified by the Reinhart and Rogoff (2009) methodology. This is a high-level analysis that does not break out the detailed composition of GDP or institutional portfolio holdings. Although GDP growth often weakened ahead of banking crises, there is nonetheless a large fraction of banking crises not preceded by weakening GDP, suggesting additional forces are at play, such as macroeconomic feedback effects. Output drops substantially in nearly all of the observed crises once stress emerges. They next use a univariate autoregressive forecasting model of GDP growth in each country, and use its worst negative forecast error as a stress scenario to be compared with the historical sample. In two-thirds of cases, the real crises were more severe than their forecasts, suggesting that care should be taken in balancing the severity-vs.-plausibility trade-off.

Another policy application of stress testing is the identification of risky or vulnerable institutions. The Supervisory Capital Assessment Program (SCAP) described by Hirtle, Schuermann, and Stiroh (2009) also applies macroeconomic scenarios—GDP growth, unemployment, and housing prices—but is more sophisticated in several important respects. First, the SCAP was a regulatory exercise to determine capital adequacy of 19 large financial institutions in the spring of 2009; the results had immediate implications for the calibration of required capital. Second, the SCAP was applied to each participating institution individually, assembling the macroprudential outcome from its microprudential parts. Third, the process included a detailed "bottom-up" analysis of the risk profile of individual portfolios and positions, using the firms' own data, models, and estimation techniques. This implies mapping from scenarios defined in terms of macroeconomic variables to the concrete inputs required by the analytics packages.

Duffie's (2011) "10-by-10-by-10" policy proposal goes a step further. Here, a regulator would analyze the exposures of N important institutions to M scenarios. For each stress scenario, each institution would report its total gain or loss against its K largest counterparty exposures for that scenario (as a rule of thumb, he suggests setting $N=M=K=10$). This would help clarify the joint exposure of the system to specific shocks, and could help identify additional important institutions via counterparty relationships to the original set of N firms. He recommends considering severe but plausible stress scenarios that are not covered by delta-based hedging and are conjectured to have potential systemic importance. He offers the

following examples, chosen to highlight broad-scope scenarios that would likely incorporate: default of a large counterparty; a 4% shift in the yield curve or credit spreads; a 25% shift in currency values or housing prices; or a 50% change in a commodities or equity-market index. As a caveat, note that many financial exposures are hedged to basis risk, which have nonlinear and non-monotonic sensitivities to risk factors, so that the magnitude of the shocks may not correlate simply with the severity of losses for a particular firm. A shortcoming of a focus on a handful of "important" institutions is the possibility of missing widely dispersed events, such as the U.S. savings and loan crisis of the 1980s.

Systemic fragility metrics supporting stress testing include Acharya, Pedersen, Philippon, and Richardson's (2010) systemic expected shortfall (SES) measure and Duffie's (2011) $10 \times 10 \times 10$ model. Chan-Lau, Espinosa, and Sole (2009) simulate their model, due to the lack of firm-level data.

2.5.3 Contemporaneous Measures: Fragility

Measuring financial fragility is not simply a matter of obtaining advance warning of impending dangers; crisis response is an important role for policymakers charged with systemic risk monitoring. Supervisory responsibilities intensify when a systemic event occurs. These tasks include ongoing monitoring of the state of the system, identification of fragile or failing institutions, markets, or sectors, the development and implementation of regulatory interventions, and clear and regular communication with the media and the public. All of this will likely need to occur within compressed time frames.

Forecasting measures that are updated on a daily or intradaily basis can be valuable as real-time signals of fragility in an emerging crisis. For example, they may clarify the possible ramifications and side effects of various interventions. A number of the models we consider can be updated frequently, including the contingent claims analysis of Gray and Jobst (2010), Adrian and Brunnermeier's (2010) CoVaR model, Adrian and Brunnermeier's (2010) and the International Monetary Fund's (2009a) related Co-Risk measures, the SES measure of Acharya, Pedersen, Philippon, and Richardson (2010), and the iPoD measures of Capuano (2008) and Segoviano and Goodhart (2009).

2.5.4 Contemporaneous Measures: Crisis Monitoring

Regardless of forecast power, some measures may still be useful in tracking a crisis as it unfolds, to aid in the allocation of staff and other resources and in the crafting of policy responses. These include the liquidity measures of Khandani and Lo (2011) and Hu, Pan, and Wang (2010), the Mahalanobis distance metric of Kritzman and Li (2010), and the absorption ratio of Kritzman, Li, Page, and Rigobon (2010). In addition, a number of the models cited above as short-horizon forecasting or fragility measures might also be deployed as contemporaneous monitoring tools; these include Adrian and Brunnermeier (2010), International Monetary Fund (2009b), Segoviano and Goodhart (2009), Capuano (2008), and Duffie (2011).

2.5.5 Ex Post Measures: Forensic Analysis

For policy purposes, measurement of the system continues to occur even after a systemic event or regulatory intervention. Publication of "flash" reports in the immediate aftermath (i.e., within hours or days) can help inform and coordinate the responses of other regulators and market participants. Such "immediate" transparency may have special significance in situations where panic or herd behavior is a factor. For example, the CFTC and SEC (2010a, 2010b) published a detailed analysis of the May 6, 2010 Flash Crash on September 30, 2010, which largely resolved the fear and uncertainty created by the unusual events surrounding that market dislocation. Imagine the market reaction if the report had been a half-hearted effort with inconsistent and inconclusive findings.

Understanding what went wrong can help in the redesign of market and regulatory practices and institutions. Borio (2010) emphasizes the critical role that measurement plays in maintaining accountability. Regulation is a repeated game, and monitoring performance can help enforce diligent behavior. In some cases, civil and/or criminal legal remedies may require thorough and unbiased explication of the sequence of events. Any of the models described above as tools for *ex ante* or contemporaneous analysis would have value as tools for *ex post* analysis. For example, Khandani, Lo, and Merton (2009) use their risk-ratcheting methodology in a historical analysis of the housing market; Getmansky, Lo, and Makarov (2004) is an *ex post* analysis of serial correlation and illiquidity in hedge fund returns.

2.5.6 Ex Post Measures: Orderly Resolution

Systemic risk analytics also have a role to play in the orderly resolution of failed institutions. This is particularly true of network models, such as Duffie (2011) or Brunnermeier, Gorton, and Krishnamurthy (2010), where a detailed understanding of the web of contractual connections can assist in the unwinding of a complex portfolio.

3   Research Perspective

In contrast to the supervisory perspective of Section 2 that involves practical challenges of implementation and policy issues, the research perspective is focused primarily on theoretical underpinnings and econometric methods. We define researchers as those skilled in developing and applying analytical techniques to economic and financial questions. As a result, the researcher's taxonomy of the systemic risk analytics surveyed in this paper is quite different from those in Tables 1–3. However, before describing this new taxonomy in more detail, we first propose a simple conceptual framework for organizing our measures of systemic risk in Section 3.1, and raise the important econometric issue of nonstationarity in Section 3.2 which is particularly relevant to systemic risk measurement. In Section 3.3, we provide a brief discussion of other research directions that are not included in this survey, but which may prove useful and bear further investigation. We then present the research taxonomy in Section 3.4.

3.1   Conceptual Framework and Econometric Issues

Denote by $\mathbf{R}_t$ the vector of asset returns of all systemically relevant entities and/or securities at date t, and let $\mathbf{X}_t$ denote the vector of state variables that capture the date-t economic and business conditions. If we define E_t to be a 0/1 indicator variable indicating the occurrence of a systemic event at date t, then the objective of any systemic risk measure is to shed light on one or more of the following three probability distributions:

$$\text{Prob}\left(\,E_t\,|\,\mathbf{R}_{t-1}, \mathbf{X}_{t-1}, \mathbf{R}_{t-2}, \mathbf{X}_{t-2}, \ldots\right) \;\equiv\; \text{Pre-Event Distribution} \tag{1}$$

$$\text{Prob}\left(\,\mathbf{R}_t, \mathbf{X}_t\,|\,E_{t-1}\,\right) \;\equiv\; \text{Post-Event Distribution} \tag{2}$$

$$\text{Prob}\left(\,\mathbf{R}_t, \mathbf{X}_t, E_t\,\right) \;\equiv\; \text{Contemporaneous Disbribution .} \tag{3}$$

The first distribution is the most relevant from the regulatory perspective: what can we say about the likelihood of a future systemic event given current and past conditions? The second is critical for determining the appropriate responses to systemic shocks. And the third is relevant for evaluating and refining our understanding of what a systemic event is.[18]

At this level of generality, (1)–(3) is nearly vacuous, but it does serve the useful purpose of motivating the need for additional structure—theoretical and econometric specifications and constraints—to narrow the parameter space of these highly nonlinear high-dimensional multivariate distributions. In particular, we must first identify the relevant institutions and securities to study ($\mathbf{R}_t$), then narrow our field of vision to a specific set of state variables ($\mathbf{X}_t$) that are relevant to the particular notion of systemic risk we wish to capture (E_t), decide on the appropriate time horizon and sampling frequency for these variables, and then formulate a suitable parametrization of the appropriate probability distribution in (1)–(3)— presumably guided by theory and practice—that is amenable to parameter estimation and statistical inference.

When described in this formulaic way, it becomes obvious that we are unlikely to ever develop a single measure of systemic risk; the dimensionality and complexity of (1)–(3) imply that multiple measures must be used to piece together a coherent, albeit incomplete, view of possible threats to financial stability. For example, if we specify the returns of publicly traded financial institutions for $\mathbf{R}_t$, and define a systemic event as simultaneous losses among multiple financial institutions, then Adrian and Brunnermeier's (2010) CoVaR, the International Monetary Fund's (2009b) Co-Risk, and Acharya, Pedersen, Philippon, and Richardson's (2010) systemic expected shortfall measures are the result. However, if our focus is on the network topology of the asset returns of the financial system, then the Granger-causality network measure of Billio, Getmansky, Lo, and Pelizzon (2010) and the absorption ratio of Kritzman, Li, Page, and Rigobon (2010) are more relevant. By narrowing

---

[18]We note two key assumptions implicit in this framework. First, since the expectations and conditioning revolve around past asset returns, we implicitly restrict attention away from data and methodologies that are not traditional financial econometrics. While financial econometrics should predominate, there are other sources of information and other techniques may warrant attention. For example, there are accounting measures (including the flow of funds data), surveys of experts and industry insiders, visual analytics, linguistic analyses (e.g., sentiment analyses of news reports), etc. Second, there is the reification of a "systemic event", which occurs at a point in time, t, since that is how systemic threats typically manifest their damage. Such a focus may discourage the analysis of threats that do not play out abruptly in calendar time. Although abrupt discontinuities are important, these are not the only outcomes to worry about. For example, Reinhart and Rogoff (2009) point to "post-event" episodes that play out in historical time (i.e., over months and years).

the set of possible free parameters for the distributions in (1)–(3), we are able to infer more precise information regarding specific aspects of systemic risk.

3.2 Nonstationarity

Even after doing the hard work of narrowing down the parameter space in (1)–(3) to yield a tractable specification that can be estimated, there is still the remaining question of how to conduct the estimation and statistical inference. Virtually all methods of estimation and inference rely on the assumption of stationarity:

$$\forall\, t_1, t_2, t_3, k: \quad \text{Prob}\left(\mathbf{R}_{t_1}, \mathbf{X}_{t_2}, E_{t_3}\right) \;=\; \text{Prob}\left(\mathbf{R}_{t_1+k}, \mathbf{X}_{t_2+k}, E_{t_3+k}\right). \tag{4}$$

In other words, the joint distribution of the relevant variables is stable over time. The motivation for such an assumption is clear: we are attempting to use historical data to infer something about the structure of systemic risk, and if that structure is not stable over time, historical data may not be an accurate guide to what the future holds. The well-known mutual-fund disclaimer that "past performance is no guarantee of future returns" can take hold with a vengeance in such circumstances.

Nonstationarity is not a new challenge to econometrics, and a large literature has developed to address specific types of nonstationarities such as deterministic and stochastic trends, and cointegration relationships.[19] However, these are very specific types of nonstationarity, whereas the kind of nonstationarity that affects systemic risk may be less easily parametrized, e.g., political, institutional, and cultural changes. In fact, the very notion of systemic risk is a good illustration of nonstationarity. Two decades ago, credit default swaps, collateralized debt obligations, ETFs, strategic mortgage defaults, and high-frequency trading would not have been part of any theoretical or empirical analysis of systemic risk. Today, they are systemically relevant markets and activities that must be carefully monitored.

The very nature of systemic risk implies a certain degree of nonstationarity that may not always be consistent with the econometric framework in which risk measures are typically estimated. While financial innovation can be useful in facing immediate challenges, it can have unintended consequences by reducing transparency and increasing complexity in the system. Significant innovations can disrupt empirical relationships, rendering reliable

---

[19]See, for example, Hamilton (1994).

statistical estimation difficult or impossible. Accordingly, the amount of data available for addressing systemic risk may be intrinsically more limited than other areas of econometric analysis. One concrete illustration of this limitation is the default probability estimates of mortgage-backed securities during the years immediately preceding the recent problems in the U.S. subprime mortgage market. A key parameter of those default probability estimates was the correlation of defaults of individual mortgages in a geographically diversified pool. Because there had been no significant national decline in the value of residential real estate in the trailing 20-year history of U.S. housing prices, the estimated default correlations were extremely low, leading to even lower default-probability estimates for the diversified pool of mortgages and higher credit ratings.

However, spotting the danger of nonstationarity is considerably easier than addressing it satisfactorily. Because nonstationarity is a vastly broader set of outcomes than its complement, the curse of dimensionality suggests that there are no easy fixes. One common approach among financial industry practitioners is to use rolling windows of data in estimating models and parameters, in some cases with exponentially declining weights to give more emphasis to current observations and less to older ones. While this practice does capture simple nonstationarities, it does so in a very crude manner that can yield other types of misleading inferences. For example, Lo and Newey (2011) show that if a time series is indeed stationary, then an exponentially weighted mean is an inconsistent estimator of the population expectation, implying that even as the sample size increases without bound, the estimator will not converge in probability but will continue fluctuating randomly. This suggests that even when economic conditions are stable, systemic risk measures estimated with exponential weights can yield "false positives" on a regular basis.

These considerations underscore the importance of incorporating realistic institutional features and constraints in modeling and measuring systemic risk, and also highlights the need for new econometric methods that are able to address nonstationarity in more sophisticated ways.

3.3 Other Research Directions

Several other research directions that we did not include in this survey may yield additional insights into systemic risk, and bear further investigation. One of the most intriguing of these "non-standard" approaches is agent-based modeling (ABM) techniques, in which economic

agents with relatively simple behavioral rules are allowed to interact freely in a computer simulation, with the objective of studying the dynamic properties of these interactions over the course of the simulation. ABM has deep intellectual roots that go back to the 1940s with John von Neumann's creation of "cellular automata".[20] The motivation is compelling: because the dynamics of realistic interactions between a large population of economic agents are far too complicated to compute analytically, simulation is a natural and efficient alternative, especially given the tremendous increase in computing power in recent years. Axelrod (1997) provides a useful introduction to this literature, and there are many online resources to help the uninitiated get started.[21] Farmer and Foley (2009) have made a compelling case for using ABM techniques in studying the financial crisis, and Farmer and colleagues have received several large grants to develop new computational models for this purpose. In addition, ABM is a topic that has engaged the interest of FSOC and OFR staff.

Another potentially relevant research area is the empirical properties of extreme returns of financial assets, i.e., "tail probabilities". Although a number of techniques in this survey do involve tail probabilities and extreme events (see, for example, Sections C.2, C.4, E.1, E.3, and E.4 of the Appendix), the "econophysics" literature—a discipline that, curiously, has been defined not so much by its focus but more by the techniques (scaling arguments, power laws, and statistical mechanics) and occupations (physicists) of its practitioners—has taken a very different tack. By carefully measuring the mathematical properties of tail probabilities of financial data, econophysicists have documented power laws that provide more accurate descriptions of how these non-Gaussian probabilities decay for more extreme scenarios. These findings have important implications for traditional risk measures such as value-at-risk and expected-loss statistics, but also imply slowly decaying autocorrelations, long-range dependence, and non-normal asymptotic distributions for most standard econometric estimators. Mantegna and Stanley (2000, 2009) provide an excellent summary of this literature, and Bouchaud, Farmer, and Lillo (2009) present a fascinating market-microstructure application of these techniques that may be particularly relevant for high-frequency trading contexts.

A third research direction that may be useful is behavioral economics and finance. This may seem contrary to the quantitative focus of systemic risk measurement, but two con-

---

[20]Cellular automata are mathematical constructions involving a simple grid of "cells" that have two states, "on" and "off", with rules for how these states evolve over time. From a relatively spare set of assumptions, these cellular automata can generate a surprisingly rich spectrum of patterns.

[21]See, in particular, `http://www2.econ.iastate.edu/tesfatsi/abmread.htm`.

siderations should give even the most skeptical readers pause in dismissing this literature. The first is the observation that among the many nonstationarities that characterize financial markets and their regulatory environment, the one constant throughout is human behavior—*Homo sapiens* has changed relatively little over the past 60,000 years. In fact, it can be argued that the ultimate source of systemic risk is the inherent incompatibility of human behavior (which has been adapted to the environment of the Neolithic ice age) with the many technological innovations of modern civilization. For example, for the first time in human history, at the click of a mouse button, we are now able to wipe out a substantial portion of our life savings with one bad trade.

The second observation is that the behavioral literature has progressed far beyond the less analytical and more phenomenological approach of the early experimental studies of behavioral biases and anomalies. Recent advances in the cognitive neurosciences have provided more formal and specific underpinnings of human behavior and their implications for financial decisionmaking,[22] and the implications for systemic risk measurement may be significant. For example, in reviewing the financial crisis from a cognitive neurosciences perspective, Lo (2011) observes that risk perception may differ from risk reality, and because the former drives behavior, not the latter, financial crises may be an inevitable outcome of free enterprise. In particular, he cites the example of the so-called "Peltzman effect" (Peltzman, 1975) in which regulations mandating the installation of various automobile safety devices may have the unintended consequence of encouraging people to drive more recklessly because they feel safer. While this effect has been challenged by a number of subsequent studies that control for various confounding factors such as enforcement practices, driver age, rural vs. urban roads, and vehicle weight, in the more narrowly defined context of NASCAR drivers, the Peltzman effect has been confirmed. This behavioral version of the Lucas critique is an ironic twist of fate in which the cognitive neurosciences are now providing neurophysiological micro-foundations for economic ideas such as rational expectations.[23] By developing a better understanding of the cognitive foundations of such patterns of behavior—including the subtleties of their context dependence—we may be able to construct more informative measures of systemic risk, as well as more responsive policies for promoting financial stability.

---

[22]See, for example, Bossaerts (2009).

[23]In fact, the "theory of mind" literature in psychology is intimately related to the formation of expectations and what economists consider to be rational behavior. See Lo (2011) for further discussion.

3.4 Research Taxonomy

Although no single classification scheme can encompass all of the relevant characteristics of all of our systemic risk measures, and there is inevitable overlap among them, from the research perspective, the taxonomy proposed in Table 4 may be more user-friendly to researchers in allowing them to identify common themes, algorithms, and data structures quickly within each category. The main differences between this taxonomy and those of Tables 1–3 stem from the fact that the origin of systemic events throughout history seem to be the four "L's" of financial crisis: liquidity, leverage, losses, and linkages. When leverage is used to boost returns, losses are also magnified, and when too much leverage is applied, a small loss can easily turn into a broader liquidity crunch via the negative feedback loop of forced liquidations of illiquid positions cascading through the network of linkages within the financial system. From this stylized narrative of financial crisis, we can categorize our systemic risk measures into five groups organized by the particular aspect of the four L's they capture and the techniques used: probabilities of loss, default likelihood, illiquidity, network effects, and macroeconomic conditions.

3.4.1 Probability Distribution Measures

Perhaps the most direct measure of systemic risk is simply the joint distribution of negative outcomes of a collection of systemically important financial institutions. The financial turbulence model of Kritzman and Li (2010), the banking system's multivariate density (BSMD) function of Segoviano and Goodhart (2009), and the co-dependence measures of Adrian and Brunnermeier (2010) (CoVaR), International Monetary Fund (2009a) (Co-Risk), and Acharya, Pedersen, Philippon, and Richardson (2010) (SES) are all examples based on the joint distribution of asset returns. These measures are largely atheoretical, but some may interpret this as a virtue rather than a vice; regardless of one's theoretical priors, these measures can still provide informative estimates of correlated losses. Moreover, the probability distributions on which these measures are based often serve as inputs to other measures with more structure. For example, Segoviano and Goodhart's (2009) BSMD is used to produce the joint probability of default (JPoD); banking stability index (BSI); distress dependence matrix (DDM); and the probability of cascade effects (PCE).

Systemic Risk Measure	Section
Probability Distribution Measures:	
Mahalanobis Distance	C.2
Multivariate Density Estimators	C.4
CoVaR	E.1
Co-Risk	E.3
Marginal and Systemic Expected Shortfall	E.4
Contingent-Claims and Default Measures:	
The Default Intensity Model	B.1
Contingent Claims Analysis	C.1
The Option iPoD	C.3
Simulating the Housing Sector	C.5
Consumer Credit	C.6
Distressed Insurance Premium	E.2
Illiquidity Measures:	
Mark-to-Market Accounting and Liquidity Pricing	B.7
Noise as Information for Illiquidity	F.3
Crowded Trades in Currency Funds	F.4
Equity Market Illiquidity	F.5
Serial Correlation and Illiquidity in Hedge Fund Returns	F.6
Broader Hedge-Fund-Based Systemic Risk Measures	F.7
Network Analysis Measures:	
Network Analysis and Systemic Financial Linkages	B.2
Granger-Causality Networks	B.5
Bank Funding Risk and Shock Transmission	B.6
Principal Components Analysis	C.7
Macroeconomic Measures:	
Costly Asset-Price Boom/Bust Cycles	A.1
Property-Price, Equity-Price, and Credit-Gap Indicators	A.2
Macroprudential Regulation	A.3
Simulating a Credit Scenario	B.3
Simulating a Credit-and-Funding-Shock Scenario	B.4
GDP Stress Tests	D.1
Lessons from the SCAP	D.2
A 10-by-10-by-10 Approach	D.3
Risk Topography	F.1
The Leverage Cycle	F.2

Table 4: Taxonomy of systemic risk measures by research method.

3.4.2 Contingent-Claims and Default Measures

With additional structure regarding an institution's assets and liabilities, it is possible to construct measures of default likelihood for each institution and then link them either directly or indirectly through their joint distribution, as in the International Monetary Fund (2009b) default intensity model. Using a nonparametric estimation technique known as "machine learning" applied to bank transactions and credit-bureau data for customers of a major U.S. commercial bank, Khandani, Kim, and Lo (2010) construct nonlinear, non-parametric, out-of-sample forecasts of consumer credit risk that significantly improve the classification rates of credit-card delinquencies and defaults.

For a more structural approach to modeling default, Merton (1973) shows that equity can be viewed as a call option on a firm's assets, and once a stochastic process for the asset's value is chosen, equity and debt contracts on those assets, and implied default probabilities, can easily be valued using contingent-claims analysis (i.e., derivatives pricing models). This is the approach taken by Capuano (2008), Gray and Jobst (2010), and Huang, Zhou, and Zhu (2009a).

Contingent claims analysis can also be applied to measuring the implicit cost of guarantees, as in Khandani, Lo, and Merton's (2009) simulation of the magnitude of cumulative losses borne by mortgage lenders through the implicit put option in non-recourse mortgages.

3.4.3 Illiquidity Measures

Illiquidity is an example of a highly specific measure of systemic risk that often requires considerable structure. Because of their role in providing maturity transformation as a valuable service, banks are vulnerable to funding illiquidity. This fragility forms the rationale for some of the main weapons in the macroprudential arsenal, including deposit insurance and the lender of last resort. These issues appear repeatedly in the literature, including recent papers by Kapadia, Drehmann, Elliott, and Sterne (2009) and Brunnermeier and Pedersen (2009). The Bank of England has developed its risk assessment model for systemic institutions (RAMSI) to simulate the possibilities (Aikman, Alessandri, Eklund, Gai, Kapadia, Martin, Mora, Sterne, and Willison, 2010). Ricks (2010) and Pozsar, Adrian, Ashcraft, and Boesky (2010) point out that funding troubles can apply to both traditional intermediaries as well as shadow banks.

Liquidity also affects the other side of the ledger. A key aspect of asset liquidity is

the valuation methods used to mark positions, either to model or to market. Sapra (2008) considers the trade-offs in the choice between these two valuation regimes, and shows benefits and costs to both. Hu, Pan, and Wang (2010) propose a measure of illiquidity by computing the deviation of observed market yields on Treasury bonds from their model-based yields derived from a daily estimate of the zero-coupon curve, and find that deviations are typically quite low (and liquidity correspondingly high), but spike during crises as arbitrage capital exits the marketplace. Pojarliev and Levich (2011) use a proprietary high-frequency dataset of currency funds' returns to capture the "crowded trade" phenomenon in currency markets. From a systemic perspective, the most interesting results arise when funding illiquidity and asset illiquidity interact to generate self-reinforcing feedback of funding shortfalls and asset fire sales, which propagate to additional funding shortfalls elsewhere. Examples include Kapadia, Drehmann, Elliott, and Sterne (2009) and Brunnermeier and Pedersen (2009).

Among the approaches described below, Khandani and Lo (2011) propose two distinct measures of equity market liquidity, one of which is the profitability of an equity mean-reversion strategy, and the other is a more direct measure of price impact based on Kyle (1985). For assets that are not publicly traded such as hedge-fund and private-equity returns, Getmansky, Lo, and Makarov (2004) propose using serial correlation as a proxy for illiquidity. By definition, current prices in illiquid markets are frequently unavailable or unreliable, forcing funds to report mark-to-model estimates that often rely on linear extrapolation pricing methods. Serial correlation in observed returns is an artifact of this autoregressive smoothing, thus providing an indication of illiquidity.

3.4.4 Network Analysis Measures

Like probability distribution measures, measures of connectedness are largely atheoretical, but they do offer more direct indications of linkages between firms, and are easily aggregated to produce overall measures of "tight coupling". One approach is to use principal components analysis to gauge the degree of commonality among a vector of asset returns. When the asset returns of a collection of entities are jointly driven by a small number of highly significant factors, fewer principal components are needed to explain the variation in the vector of returns, hence sharp increases in the proportion of variability explained by the first n principal components is a natural indication of systemic risk. The absorption ratio of Kritzman, Li, Page, and Rigobon (2010) and the PCAS measure of Billio, Getmansky, Lo,

and Pelizzon (2010) are based on this property.

More explicit measures of financial networks may be derived from graph theory, a branch of discrete mathematics in which abstract "nodes" are connected to each other by "edges" that represent a particular type of relationship. Such networks have been popularized through social networking websites and degree-of-separation games, but there is a rich set of analytics that have been developed for networks which can be drawn upon to measure systemic risk. Chan-Lau, Espinosa, and Sole (2009) and the International Monetary Fund (2009b) contain two network models of interbank exposures to assess the network externalities of a bank failure using institutional data. Using Granger-causality test statistics for asset returns to define the edges of a network of hedge funds, banks, broker/dealers, and insurance companies, Billio, Getmansky, Lo, and Pelizzon (2010) show that Granger-causality networks are highly dynamic and become densely interconnected prior to systemic shocks.

And the funding gap model of Fender and McGuire (2010a) reveals important linkages within multinational banks that have many geographically dispersed offices. While aggregate balance sheet data at the banking-group level may not show much risk, a network map of the exposures between offices within a banking group may yield a very different picture, especially for large banking organizations that fund their local foreign currency (especially USD) positions through their internal (i.e., within the banking group) and external networks.

3.4.5 Macroeconomic Measures

The diametric opposite of the atheoretical probability-distribution measures of Section 3.4.1 are the macroeconomic models of systemic risk. Because the macroeconomy is so complex, it is virtually impossible to derive useful information from basic macro data without significant structural hypotheses. Accordingly, there are a multitude of macroeconomic measures of systemic risk, corresponding to the many macro models of business and credit cycles, unemployment, inflation, and growth.

The comprehensive volume by Reinhart and Rogoff (2009) provides useful comparisons of broad macroeconomic aggregates such as asset price indices (equities, housing, etc.), GDP growth rates, and public debt over many financial crises, and find a number of common patterns. Alfaro and Drehmann (2009) use the Reinhart and Rogoff episodes as their starting point for generating GDP stress tests.

A natural complement to systemic risk measurment is macroprudential regulation, which

Borio (2010) defines as calibrating supervision from the top down, rather than building it up from supervision of individual institutions. Caruana (2010b) makes the case for countercyclical regulation, arguing that if Basel III had existed at the time of the crisis, banks would have had much stronger capital bases so that the negative feedback from credit losses to credit supply—i.e., procyclical aggravation of the business cycle from financial distress— would have been milder, and the required bailouts much smaller.

Alessi and Detken (2009) construct simple early-warning indicators from a broad range of real and financial indicators—including GDP and its components, inflation, interest rates, and monetary aggregates—for 18 OECD countries between 1970 and 2007. Extreme values of these aggregates are taken as indications of pending booms or busts over the following 6-quarter horizon. Borio and Drehmann (2009b) propose a related approach, but with signals defined by simultaneous extreme values for pairs of property prices, equity prices, and credit spreads, again drawn from 18 industrialized countries between 1970 and 2007.

4   Data Issues

While this survey covers a diverse range of models of threats to financial stability, they all have one feature in common: significant new data requirements. Although there is still considerable controversy over the root causes of the Financial Crisis of 2007–2009, there is little dispute that regulators, policymakers, and the financial industry did not have ready access to information to generate early warning signals or implement rapid resolution plans. For example, prior to the Dodd Frank Act, even systemically important financial institutions such as AIG and Lehman Brothers were not obligated to report their amount of financial leverage, asset illiquidity, counterparty risk exposures, market share, and other critical risk data to any regulatory agency. If aggregated over the entire financial industry, such data could have played a crucial role in providing regulators with advance notice of AIG's unusually concentrated position in credit default swaps, and the broad exposure of money market funds to Lehman bonds.

The Dodd Frank Act mandates central reporting of large swaths of the over-the-counter (OTC) derivatives market, and has assigned to the OFR and FSOC the responsibility for coordinating data collection, data sharing, and supervision of financial firms. Similar efforts are underway in Europe, with the creation of the European Systemic Risk Board (ESRB). The Financial Stability Board (FSB) and International Monetary Fund (IMF) are spearheading

an effort for the G-20 finance ministers and central bank governors to address information gaps at the international level (see Financial Stability Board and International Monetary Fund (2010)). These efforts will undoubtedly raise many new issues surrounding data acquisition, archiving, and management. In this section, we provide a brief introduction to some of these issues by summarizing in Section 4.1 the data required by the risk analytics in this survey, reviewing the issues surrounding the standardization of legal entity identifiers (LEIs) in Section 4.2, and discussing recent advances in computer science that have significant implications for the trade-off between transparency and privacy in Section 4.3.

4.1 Data Requirements

To be able to implement the statistical models and algorithms for calculating various systemic risk measures described in this paper, risk regulators will have to collect, archive, and access data on a regular basis, while addressing security and privacy concerns of all stakeholders. To provide a concrete illustration of the scope of this effort, we provide in Table 5 a detailed list of the data sources used by the measures in this survey.

4.2 Legal Entity Identifier Standards

Separately, the OFR and FSB are coordinating the development of a standardized legal entity identifier (LEI) registry, which would, for the first time, provide consistent global identification of obligors in financial transactions. The LEI has special significance for systemic risk measurement because it facilitates the implementation of many of the network measures described in this survey.

The need for a standardized method of identification is easiest to see within—but not limited to—the context of network or graph-theoretic measures such as Chan-Lau, Espinosa, and Sole (2009) and Duffie (2011), where the nodes in the graph represent legal entities, and edges represent individual or aggregated contractual relationships. In practical implementations of such models, especially with systemic scope, both entities and relationships will be first-class objects with persistent state. This fact implies a need for an efficient, consistent, globally unique identification scheme for both entities and relationships. An LEI is simply a systematically maintained tag or code that uniquely identifies an entity in the system. Bottega and Powell (2010) describe LEIs in detail, noting that they are "a critical component in measuring and monitoring systemic risk", because they enable the construction of

Table 5: Data Requirements

Replicable Studies Covered in this Document

Source	Asset	Type	Frequency	Start	Stop
Asset Price Boom/Bust Cycle					
Alessi and Detken (2009)					
IMF International Financial Statistics	Credit	Price	Quarterly	Q1:1970	Q4:2007
OECD Econ Outlook, Main Econ Indicators	Macro	Macro	Annual	Q1:1970	Q4:2007
OECD Econ Outlook, Main Econ Indicators	Macro	Macro	Quarterly	Q1:1970	Q4:2007
BIS and ECB sources	Money Market	Return	Quarterly	Q1:1970	Q4:2007
BIS	Real Estate	Price	Quarterly	Q1:1970	Q4:2007
Bank Funding Risk and Shock Transmission					
Fender and McGuire (2010b)					
BIS locational banking statistics by nationality	Bank	Accounting	Quarterly	Q1:2000	Q1:2010
BIS locational statistics by residency	Bank	Accounting	Quarterly	Q1:2000	Q1:2010
Consumer Credit					
Khandani, Kim, and Lo (2010)					
Proprietary Commercial Bank data	Bank	Miscellaneous	Monthly	January 2005	April 2009
Contingent Claims Analysis					
Gray and Jobst (2010)					
Moody's KMV creditEdge	Bond	Price	Quarterly	January 1, 2007	January 1, 2010
Bloomberg	Option	Price	Daily	January 1, 2007	January 1, 2010
MarkIt	Swap	Spread	Daily	January 1, 2007	January 1, 2010
Co-Risk					
International Monetary Fund (2009a)					
FRB H15 Release	Bond	Return	Daily	July 1, 2003	September 12, 2008
CRSP	Equity	Return	Daily	July 1, 2003	September 12, 2008
Bloomberg	Money Market	Return	Daily	July 1, 2003	September 12, 2008
FRBNY Website	Money Market	Return	Daily	July 1, 2003	September 12, 2008
CBOE Website	Option	Return	Daily	July 1, 2003	September 12, 2008
Bloomberg and Primark Datastream	Swap	Price	Daily	July 1, 2003	September 12, 2008

Data Requirements (cont.)

CoVAR
Adrian and Brunnermeier (2010)

FRBNY Website	Bond	Return	Weekly	Q1: 1986	Q1: 2010
CRSP	Equity	Price	Weekly	Q1: 1986	Q1: 2010
COMPUSTAT	Equity	Accounting	Quarterly	Q1: 1986	Q1: 2010
Bloomberg	Money Market	Return	Weekly	Q1: 1986	Q1: 2010
FRB H15 Release	Money Market	Return	Weekly	Q1: 1986	Q1: 2010
CBOE Website	Option	Return	Weekly	Q1: 1986	Q1: 2010
CRSP	Real Estate	Return	Weekly	Q1: 1986	Q1: 2010

Crowded Currency Trades
Pojarliev and Levich (2011)

DB Currency Harvest G10 Index	Currency	Return	Weekly	April 2005	June 2010
AFX Currency Mgmt. Index	Currency	Return	Weekly	April 2005	June 2010
DB Currency Volatility Index	Currency	Return	Weekly	April 2005	June 2010
DB G10 Valuation Index	Equity	Return	Weekly	April 2005	June 2010
Proprietary	Private Partnership	Return	Weekly	April 2005	June 2010

Default Intensity
Giesecke and Kim (2009)

Moody's Defaults Risk Services	Bond	Ratio	Static	January 1, 1970	December 31, 2008

Distressed Insurance Premium
Huang, Zhou, and Zhu (2009a)

Market Participants	Bond	Probability	Weekly	January 2001	December 2008
Moody's KMV	Bond	Probability	Weekly	January 2001	December 2008
FRB H15 Release	Bond	Spread	Weekly	January 2001	December 2008
TAQ Database	Equity	Price	Tick	January 2001	December 2008
Bloomberg	Equity	Return	Quarterly	January 2001	December 2008
FRB H15 Release	Money Market	Return	Weekly	January 2001	December 2008
Bloomberg	Option	Implied Vol	Weekly	January 2001	December 2008
MarkIt	Swap	Spread	Daily	January 2001	December 2008

Early Warning Macro Indicators
Borio and Drehmann (2009b)

BIS	Macro	Price	Annual	1970	2007
BIS	Macro	Miscellaneous	Static	1970	2007

Equity Market Liquidity
Khandani and Lo (2011)

NYSE Trade and Quote (TAQ)	Equity	Price	Tick	July 2, 2007	September 28, 2007

GDP Stress Tests					
Alfaro and Drehmann (2009)					
Authors' Estimates	Macro	Miscellaneous	Static	1970:Q1	2007:Q4
BIS	Macro	Return	Quarterly	1970:Q1	2007:Q4
Granger Causality Networks and PCA					
Billio, Getmansky, Lo, and Pelizzon (2010)					
CRSP	Equity	Return	Monthly	January 1994	December 2008
CRSP/Compustat Merged	Equity	Accounting	Quarterly	January 1994	December 2008
CS/Tremont hedge fund index	Private Partnership	Return	Monthly	January 1994	December 2008
TASS Database	Private Partnership	Return	Monthly	January 1994	December 2008
TASS Database	Private Partnership	Ratio	Monthly	January 1994	December 2008
Hedge Fund Based Systemic Risk Measures					
Chan, Getmansky, Haas, and Lo (2006b, 2006b)					
TASS Database	Private Partnership	Return	Monthly	January 1977	August 2004
TASS Database	Private Partnership	Return	Mixed	January 1994	August 2004
CSFB/Tremont Hedge Fund category indices	Private Partnership	Return	Monthly	January 1994	August 2004
Housing Sector					
Khandani, Lo, and Merton (2009)					
CRSP	Bond	Return	Monthly	February 1977	December 2008
Robert Shiller Website	Bond	Return	Annual	January 1919	January 1977
SandP/Case-Shiller Home Price Composite	Real Estate	Return	Monthly	January 1987	December 2008
FHFA national house price index	Real Estate	Return	Quarterly	Q1: 1975	Q4: 1986
Nominal home price index collected by R. Shiller	Real Estate	Return	Annual	1919	1974
U.S. Census Bureau	Real Estate	Number	Monthly	January 1963	December 2008
U.S. Census Bureau	Real Estate	Number	Quarterly	1974	December 2008
U.S. Census Bureau	Real Estate	Price	Monthly	January 1963	December 2008
Freddie Mac	Real Estate	Return	Monthly	April 1971	December 2008
Mahalanobis Distance					
Kritzman and Li (2010)					
Not Specified by Authors	Bond	Return	Monthly	January 1973	December 2009
Not Specified by Authors	Commodities	Return	Monthly	January 1973	December 2009
S&P 500	Equity	Return	Monthly	January 1973	December 2009
MSCI non-U.S. Index	Equity	Return	Monthly	January 1973	December 2009
Not Specified by Authors	Real Estate	Return	Monthly	January 1973	December 2009

<div align="center">Data Requirements (cont.)</div>

Marginal and Systemic Expected Shortfall
Acharya, Pedersen, Philippon, and Richardson (2010)

CRSP	Equity	Return	Daily	June 2006	June 2007
CRSP/Compustat Merged	Equity	Accounting	Static	June 2007	June 2007
MarkIt	Swap	Spread	Daily	June 2006	June 2007

Multivariate Density Estimator
Segoviano and Goodhart (2009)

MarkIt	Swap	Spread	Daily	January 2005	October 2008

Network Analysis of Linkages
International Monetary Fund (2009b)

BIS Intl. Banking Statistics	Mixed	Price	Static	March 2008	March 2008

Noise as Information for Illiquidity
Hu, Pan, and Wang (2010)

CRSP Daily Treasury Database	Bond	Price, Return	Daily	January 1987	December 2009

Option iPoD
Capuano (2008)

Bloomberg	Option	Price	Daily	February 12, 2008	June 21, 2008

Principal Components
Kritzman, Li, Page, and Rigobon (2010)

MSCI U.S. Index and 51 Subindices	Equity	Return	Daily	January 1, 1998	January 31, 2010
For each country, the MSCI Index and all subindices	Equity	Return	Daily	January 1, 1998	January 31, 2010
Case-Shiller U.S. Housing Price Index	Real Estate	Return	Monthly	January 1987	December 2009

Serial Correlation and Illiquidity in Hedge Fund Returns
Getmansky, Lo, and Makarov (2004)

CRSP	Equity	Price, Return	Monthly	November 1977	January 2001
TASS Database	Private Partnership	Return	Monthly	November 1977	January 2001
CSFB/Tremont Hedge Fund category indices	Private Partnership	Price, Return	Monthly	November 1977	January 2001

10-by-10-by-10
Duffie (2011)

Bank Executive Survey	Mixed	Text	Quarterly	N/A	N/A

the counterparty network graph of linkages and interrelationships in the system. This is the foundation of network analysis, as described in Section B, and allows for efficient and accurate aggregation when searching for concentrated exposures and patterns of activity.

A move toward a globally standardized LEI is already underway, and the OFR is helping to coordinate an international agreement around a standardized global registry of LEIs.[24] A registry of globally unique LEIs has ancillary benefits for the financial industry, which currently replicates this costly function at each firm to support internal trading, compliance, and risk management functions.[25]

The set of instrument types defines the available contractual relationships within the system—the attributes of the edges between nodes in a counterparty network graph. By extension, the full set of instrument types establishes the universe of possible portfolios for market participants. Because there are so many possible contracts, this universe is very large indeed. The portfolio for a given participant at a particular point in time can be represented by a vector of numbers, namely the amounts of each contract type contained in the portfolio. This vector will have many elements, i.e., it will be very high-dimensional. Moreover, for most participants, it will be very sparsely populated, i.e., it will have zeroes in most elements, since most participants have relatively specialized activities. Measuring financial contracts will require the capture of much more detail about those contracts than is the case under traditional firm-centric accounting systems.

To implement forward-looking risk metrics, the goals should be to capture and understand each contract's implied cashflow commitments between the counterparties to the contract, noting that, in many cases, these cashflows are contingent on other factors. The ability to work directly with the cashflows is crucial because, in practice, it is possible for two contracts or portfolios to generate substantially identical cashflow patterns, even when their

---

[24]See Office of Financial Research (2011, 2010) for further details.

[25]In November 2010, the Office of Financial Research (2010) issued a policy statement to promote the development of a global LEI system. This included requirements for attributes of an LEI standard and associated reference data, as well as operational attributes for a system to issue and maintain LEIs. Simultaneously, the SEC and CFTC issued "Notices of Proposed Rulemaking" for reporting swap transactions to trade repositories, and expressed a preference for using an LEI for swap reporting. In January 2011, the International Organization for Standardization (ISO) launched a process to establish an LEI standard. It developed a draft specification for the standard and selected a registration authority to oversee assignment of LEIs: SWIFT, which is partnering with DTCC and its subsidiary Avox as facilities managers. The initial vote on the LEI standard (17442) being developed by ISO closed at the end of June. In September 2011, the Financial Stability Board (FSB) met in Basel to consider options for coordination around governance of a global LEI infrastructure.

legal or accounting representations differ widely. Much of financial engineering is devoted to repackaging a fixed set of cashflow commitments into a different contractual configuration, perhaps to manage or lay off risk, avoid taxable events, reduce the market impact of a trade, or simply to obfuscate the activity.[26]

4.3 Privacy vs. Transparency

Historically, government policy has tread carefully on the financial industry's disclosure requirements because much of the industry's data are considered highly proprietary. Apart from the obvious privacy issues surrounding customer financial data, the majority of intellectual property in the financial industry consists of trade secrets. Unlike other industries in which intellectual property is protected by patents, the financial industry consists primarily of "business processes" that the U.S. Patent Office deems unpatentable, at least until recently.[27] Accordingly, trade secrecy is the preferred method by which financial institutions protect the vast majority of their intellectual property, hence their desire to limit disclosure of their business processes, methods, and data. Forcing a financial institution to publicly disclose its proprietary information—and without the quid pro quo of 17-year exclusivity that a patent affords—will obviously discourage innovation.

Nevertheless, the recent crisis, as well as the skepticism with which the financial industry has greeted current proposals for systemic-risk surcharges, provide even greater motivation for the OFR's mandate to collect data from SIFIs and conduct thorough empirical analysis on the efficacy of various analytics for capturing systemic risk.

These two seemingly irreconcilable objectives—protecting trade secrets while providing regulators with systemic risk transparency—are not as difficult to reconcile as they may appear. In particular, the banking industry already provides a significant amount of proprietary data to its regulator (the Office of the Comptroller of the Currency) without jeopardizing its intellectual property, hence some of these procedures may be applied to SIFIs not currently regulated as banks. However, an even more significant development for systemic risk management is the recent breakthroughs in cryptography that enable individuals to

---

[26]Note that the Dodd Frank Act (see especially section 153(c)(2)) mandates that the FSOC standardize data types and formats for reporting. Separately, the Committee on Payment and Settlement Systems (CPSS) and International Organization of Securities Commission (IOSCO), at the direction of the Financial Stability Board (FSB), established a task force to define requirements for reporting and aggregation of over-the-counter (OTC) derivative information.

[27]See, for example, Lerner (2002).

maintain the privacy of their data through encryption algorithms that allow third parties to compute aggregate statistics across multiple individuals while preserving the privacy of each individual.[28] These algorithms will permit regulators to compute systemic risk exposures without ever requiring individual institutions to reveal their proprietary data; only encrypted information is used by the regulators. Although still in experimental stages of development, these so-called "secure multi-party computational" and "fully homomorphic encryption" algorithms will likely revolutionize the way in which systemic risk is measured and managed.

5 Conclusions

Regulators have been given a mandate by the Dodd Frank Act to measure and monitor systemic risk. Market participants have a complementary and immediate interest in better measurement and management of systemic risk. Although the impact of systemic events is widely felt, the burden for measuring and monitoring financial stability falls first and foremost on government regulators, given the unavoidable conflicts of interest faced the private sector. Because systemic risk is a multifaceted problem in an ever-changing financial environment, any single definition is likely to fall short, and may create a false sense of security as financial markets evolve in ways that escape the scrutiny of any one-dimensional perspective.

The scholarly literature is instructive in this regard. A wide variety of measurement techniques have been proposed and implemented, attempting to capture systemic risk from diverse perspectives. Ultimately, the specific measures regulators choose to deploy will become the *effective* operational definition of systemic risk, and these metrics should be chosen to tackle the problem from many different directions.

The data requirements to support these metrics are correspondingly wide-ranging. In many cases, academic researchers have made do with publicly available data, adjusting their modeling approaches accordingly. This is a constraint that regulators will not necessarily face, given the mandates and authorities granted to them by recent legislation. While the scholarly literature serves as a useful introduction to the scope of possible measurement approaches, it should be regarded only as a starting point, not a conclusion. We hope this survey will expedite the process of discovery and innovation in systemic risk measurement,

---

[28]See, for example, Abbe, Khandani, and Lo (2011).

and look forward to future editions as more stakeholders engage in this important research endeavor.

Appendix

Note On Attribution: The main goal of this appendix is to present—faithfully—in one place and in a consistent format the models and methods developed by other researchers. To this end, in the descriptions below we frequently excerpt language from the original publications. While we explicitly cite all excerpted works, we do not typically insert quotation marks or specific footnotes to avoid cluttering the text.

A   Macroeconomic Measures

The momentousness of threats to financial stability implies that genuine systemic events should be apparent even at the highest levels of aggregation. The recent book by Reinhart and Rogoff (2009) examines broad macroeconomic aggregates, such as asset price indices (equities, housing, etc.), GDP growth rates, and public debt. They spotlight the irony of common patterns of empirical dynamics of crisis episodes, repeated reliably across countries and over time. Others exploit the same set of crises; for example, Alfaro and Drehmann (2009) use the Reinhart and Rogoff episodes as their starting point for generating GDP stress tests, described in more detail below. As a matter of regulatory policy, aggregate measures may help to identify large-scale imbalances, and suggest strategic policy adjustments to address them. Reinhart and Rogoff's approach also underscores the usefulness of purely historical analyses for guiding regulatory policy.

From a policy perspective, a natural counterpart to macroeconomic risk monitoring is macroprudential regulation. Borio (2010) defines a macroprudential framework as calibrating supervision from the top down, rather than building it up from supervision of individual institutions. This perspective can help resolve a fallacy of composition, if risk is endogenous to the system while nonetheless appearing exogenous to individual firms. It is helpful to think of both the evolution of risk in a time dimension (e.g., procyclicality) and the allocation of risk in a cross-sectional dimension (e.g., common exposures and interlinkages). Per Borio (2010), the most appropriate risk measures differ for these two dimensions. Early warning systems or leading indicators are best for addressing the time dimension, while some robust measure of each institution's contribution to systemic risk is appropriate for the cross-sectional dimension. Currently, no single measure can do both simultaneously, and in fact, many extant measures can be misleading. He argues that policy implementation should push rules as far as possible, to counteract the pressures of political economy and the temptation to discount risks. However, rules should be simple to understand and tolerate a degree of discretion.

Caruana (2010b) argues that if Basel III had existed at the time, banks would have faced the crisis with much stronger capital bases, so that the negative feedback from credit losses to credit supply—i.e., procyclical aggravation of the business cycle from financial distress—would have been milder, and burden on taxpayers smaller. One aspect of Basel III is a countercyclical capital buffer based on the observation that extraordinary private credit growth often ends in widespread defaults.[29] In an earlier version, Basel Committee on

---

[29]Such a buffer can also serve as a signal for further action. For example, the widening credit and housing price gaps in the U.S., U.K. and Spain during the 2000s would have pushed a countercyclical buffer against

Banking Supervision (2010) emphasized the ratio of credit to GDP as a system-wide benchmark for accumulating buffers, which would be encouraged through restrictions on capital distributions. The virtue of this variable is mainly the fact that it can be computed in nearly every country, making cross-country comparisons and coordination easier. However, there is significant skepticism, especially among central bankers and regulators, around measures that may be too esoteric for policymakers charged with making politically unpopular decisions. If systemic threats cannot be credibly communicated to political representatives of the general population, preemptive action becomes much harder to justify.

More recently, emphasis has shifted instead on an "expected impact" approach based on a firm's "systemic footprint," which proponents argue is more amenable to international negotiation; see Basel Committee on Banking Supervision (2011). At the same time, it is noteworthy that the BIS continues to overload capital requirements with a multitude of risk-exposure limits, including market, credit, and operational risk—and now systemic too. Given that both capital and the relevant risks are all measured with error—indeed errors with potentially procyclical correlation—there is a danger of unintended consequences and a false sense of security arising from this process.

Applying macroeconomic data for early-warning models, Alessi and Detken (2009) consider a broad range of real and financial indicators—including GDP and its components, inflation, interest rates, and monetary aggregates—for 18 OECD countries between 1970 and 2007. The model emits a yes/no warning when the given indicator exceeds a specific percentile of its own distribution, forecasting an incipient boom/bust cycle (or lack thereof) over an upcoming 6-quarter horizon. An inaccurate forecast implies either a Type I or Type II error, and a simple linear mixing variable parameterizes regulators' choice of preferred (i.e., least-loss) error. This affects the outcome of "best" indicator, which the authors conclude is the "global gap"—i.e., the deviation from GDP, after accounting for trends—of either narrow money (M1) or private credit.[30] Borio and Drehmann (2009b) propose a related approach, but with signals defined by simultaneous extreme values for bivariate pairs of indicators. The pairings are the three possible combinations of a property price gap, an equity price gap, and a credit gap, again drawn from 18 industrialized countries between 1970 and 2007.[31] The authors conclude that best performance occurs for a credit gap exceeding 6%, combined with either an equity gap over 60% or a property gap exceeding 15-25%.

A.1 Costly Asset-Price Boom/Bust Cycles

Alessi and Detken (2009) use a signaling methodology to predict costly aggregate asset price boom/bust cycles. The performance of a host of real and financial variables as early warning indicators for costly aggregate asset price boom/bust cycles are examined, using data for 18

its upper bound for several years. Allowable loan-to-value ratios might have been lowered, or mortgage-interest deductions could have been limited. In the denouement of a boom, the release of the countercyclical buffer may further ease financial strains, although the timing of release is not as critical as the timing of accumulation, since the latter directly affects the amplitude of the boom and bust.

[30]Specifically, they define the gap as the detrended ratio of the variable of interest (M1, private credit, etc.) relative to GDP.

[31]They define "extreme" values as threshold exceedances, and calibrate the model for the most informative thresholds. Specifically, the signaling thresholds for triggering a warning are chosen by a grid search that optimizes a signal-to-noise ratio calculated from Type I and Type II errors.

OECD countries between 1970 and 2007. A signaling approach is used to predict asset price booms that have relatively serious real economy consequences. The authors propose a loss function to rank the tested indicators given policymakers' relative preferences with respect to missed crises and false alarms. The paper analyzes the suitability of various indicators as well as the relative performance of financial-versus-real, global-versus-domestic, and money-versus-credit-based-liquidity indicators.

A.1.1   Definition

With respect to deciding on what is an acceptable performance for an indicator, the authors decide to take into account the regulator's relative aversion with respect to Type-I and Type-II errors. A warning signal is issued when an indicator exceeds a threshold, here defined by a particular percentile of an indicator's own distribution. Each quarter of the evaluation sample for each indicator falls into one of the following quadrants of Table A.1 below:

	Costly Boom/Bust Cycle Within 6 Quarters	No Costly Boom/Bust Cycle Within 6 Quarters
Signal Issued	A	B
No Signal Issued	C	D

Table A.1: The "confusion" matrix for the signals.

The loss function used to analyze the usefulness of the indicators is:

$$L = \theta \frac{C}{A+C} + (1-\theta)\frac{B}{B+D} \tag{A.1}$$

where θ is the parameter revealing the policymaker's relative risk aversion between Type-I and Type-II errors. The usefulness of an indicator can then be defined as:

$$\min[\theta; 1-\theta] - L . \tag{A.2}$$

Note that a regulator can always realize a loss of $\min[\theta; 1-\theta]$ by disregarding the indicator. Thus, an indicator is useful to the extent that it produces a loss lower than $\min[\theta; 1-\theta]$ for a given θ.

In order to test their signals, the boom/bust cycles need to be defined. The authors use data for 18 OECD countries between 1970:Q1 and 2007:Q4.[32] The real aggregate asset price indexes have been provided by the BIS and are weighted averages of equity prices and residential and commercial real estate prices, and are deflated by the appropriate national consumption deflators.

An aggregate asset price boom is defined as a period of at least three consecutive quarters, in which the real value of the index exceeds the recursive trend plus 1.75 times the recursive standard deviation of the series. The recursive trend is calculated with a very slowly adjusting

---

[32]The countries are Australia, Belgium, Canada, Switzerland, Germany, Denmark, Spain, Finland, France, the United Kingdom, Ireland, Japan, the Netherlands, Norway, New Zealand, Sweden, the United States.

Hodrick-Prescott filter ($\lambda = 100,000$) taking into account only data up to the respective quarter. The authors then differentiate between aggregate asset price booms, which have little consequences for the real economy and those that have significant effects. A high-cost boom (HCB) is a boom, which is followed by a three year period in which overall real GDP growth has been at least three percentage points lower than potential growth. In this way, their sample of 45 classifiable booms is split into 29 high-cost and 16 low-cost booms.

They test a set of 18 real and financial variables and up to 6 different transformations of these variables, resulting in a total of 89 indicators, on their suitability as early warning indicators for high-cost asset price boom/bust cycles within a 6-quarter forecasting horizon (see Table A.2 for the list of variables).

Variable Category	Variable
Economic Variables	GDP, consumption, investment, housing investment, CPI
Financial Variables	CPI deflated equity, housing prices, the term spread
	real exchange rates, real & nominal 3-month interest rates
	real & nominal 10-year bond yields, real M1, real M3
	real private and domestic credit
GDP (PPP) weighted	private credit to GDP, M1/GDP, M3/GDP, nominal short rates
global variables	VAR shocks for M1, M3, private credit growth

Table A.2: The categories of variables tested as signals.

Note that the real money and credit growth rates have been corrected from endogenous business-cycle and asset-price components by means of recursive vector-autoregression (VAR) models.[33] Furthermore, all other variables, except aggregate asset prices, equity prices, exchange rates, and interest rates, are seasonally adjusted.

The evaluation period the authors used was 1979:Q1 to 2002:Q1. They start evaluating in 1979 because some starting window is required to compute reasonable initial trend estimates. They do not evaluate the last boom wave because they did not have three years of post-boom GDP data.

There are two main algorithms that the authors apply:

- **Individual Country Case:** For this case, optimizing the objective function in (A.2) is as follows: for a given θ, for a given indicator, and for a given country, a grid search is done to find the best percentile in the range $[0.05, 0.95]$ which maximizes (A.2). Note that although this optimal percentile is derived *ex post* by using all available high-cost booms per country, the threshold varies over time as the percentile is applied to quarterly updated distributions of the indicator as time passes.

- **Individual Indicator Case:** For this case, optimizing the objective function in (A.2) is as follows: for a given indicator and a given θ, a grid search is done to find the best *common* percentile in the range $[0.05, 0.95]$ across countries; that is, the common percentile chosen is the one that minimizes the aggregate loss over all countries.

---

[33]Note the subtle but important distinction between the acronym "VAR", which denotes "vector autoregression", and "VaR", which denotes "value-at-risk".

When computing the resulting figures for A–D of the matrix shown in Table A.1, the authors exclude boom periods as of the fourth consecutive quarter of which a signal is present because by then, a warning signal is no longer useful.

Apart from the usefulness metric in (A.2), one can also compute the true positives (tp) and the true negatives (tn) for each signal:

$$tp = \frac{A}{A + B} \tag{A.3}$$

$$tn = \frac{D}{C + D} \tag{A.4}$$

Furthermore, one can compute the difference between the conditional (on a signal being issued) and unconditional probabilities of the event:

$$dp = \frac{A}{A + B} - \frac{A + C}{A + B + C + D} . \tag{A.5}$$

The larger dp is, the more valuable an indicator it is. Another useful statistic is the average lead time (ALT) of an indicator, which is defined as the average number of leading quarters which an indicator has been signaling an event for the first time.

A.1.2 Inputs and Outputs

- **Input:** The quarterly time series for the candidate indicators (presented in Table A.2) between 1970:Q1–2007:Q4. All time series except those for asset prices are seasonally adjusted. Furthermore, the real money and credit growth rates have been corrected from endogenous business cycle and asset price components by means of recursive VAR models.[34]

  Furthermore, a publication lag of 1 quarter is assumed, meaning that an indicator is calculated for each quarter with variables lagged by 1 quarter.

  With respect to the sources of their data, the authors are not too specific as some of it is not public. According to them, the main data source is the *OECD Economic Outlook and Main Economic Indicators*. Domestic and private credit are obtained from the *IMF International Financial Statistics*, lines 32 and 32D, respectively. Asset price indexes were provided by the BIS. Narrow monetary aggregates are from BIS and ECB sources.

- **Input:** The regulator's θ. The authors run their analysis for the following: $\theta = (0.2, 0.3, 0.4, 0.5, 0.6, 0.8)$.

- **Output:** The true positives and true negatives (equations (A.3) and (A.4), respectively).

---

[34]See Adalid and Detken (2007) for a description of the methodology to derive the shocks used to correct these variables. The VARs are estimated recursively to mimic real-time data availability and 6 quarters of moving averages of the derived shocks are used.

- **Output:** The usefulness of each indicator as defined in (A.2).

- **Output:** The dp as defined in (A.5).

- **Output:** The average lead time (ALT) of an indicator, defined as the average number of leading quarters which an indicator has been signaling an event for the first time.

A.1.3 Empirical Findings

The authors find that the global M1 gap and the global private credit gap are the best early warning indicators for costly boom/bust cycles. Interestingly, these indicators are global variables, which can be explained by the fact that asset price boom/bust cycles are largely international phenomena. However, the "goodness" of an indicator depends on the θ of the policymaker; the usefulness of the approach is not breathtaking when policymakers have a clear preference for either Type-I or Type-II errors. In the case of relatively balanced preferences, the best indicator reduces the preference-weighted sum of Type-I and Type-II errors by as much as 25% compared to a situation in which the indicator is ignored.

The best indicator for a policymaker who is only slightly more averse to false alarms than missed crises is the global private credit gap. In terms of the absolute performance, using the estimated optimal 70% common percentile across countries had a true positive rate of 82%. Moreover, the average lead time for the first warning signal was 5.5 quarters.

Overall, the authors find that financial variables contain more information for predicting costly asset price booms than real indicators, that global financial indicators perform better than domestic ones, and that global credit outperforms global money.

The signaling approach as described in their paper is presented in Kaminsky, Lizondo, and Reinhart (1998), and used to predict foreign exchange and banking crises in Borio and Lowe (2004) and Borio and Drehmann (2008); these other authors use the signal-to-noise ratio to determine the usefulness of a signal as opposed to the "usefulness" index in equation (A.2).

A.2 Property-Price, Equity-Price, and Credit-Gap Indicators

Borio (2009) construct macroeconomic early warning indicators to predict banking sector crises by extending the framework of Borio and Lowe (2004). The three indicators used are the property price gap, the (real) equity price gap, and the credit gap. This approach is grounded in the endogenous-cycle view of financial instability. The authors argue that the coexistence of unusually rapid credit growth and asset prices indicate the build-up of financial imbalances that raise the likelihood of subsequent financial distress. This approach is similar to that of Alessi and Detken (2009) which is summarized in Section A.1.

A.2.1 Definition

To define the equity-price, property-price, and credit-gap indicators, trends need to be estimated. For all three time series, the authors use one-sided Hodrick-Prescott (HP) filters with $\lambda = 1{,}600$, which employ information available only up to the time when the predictions are made. Using the gaps of the three variables as indicators, the authors try to predict crises

that occur within 3 years of a set of signals breaching some threshold. The authors define a country crisis in two ways:

1. If one or more large banks in a given country failed or had to be supported by the government in an emergency operation.

2. If the country undertook at least one of the following policy operations: extended deposit insurance guarantees beyond the household sector, purchased assets, or injected capital.

The authors use two objective functions in defining the optimal indicator thresholds. One objective is to minimize the noise-to-signal ratio (nts), which is defined as:

$$nts \quad = \quad \frac{\text{Type-II errors}}{1 - \text{Type-I errors}} \; . \tag{A.6}$$

The second objective is to maximize the number of crises predicted, i.e., the true positive rate.

The authors are interested in assessing the performance of joint signal indicators, meaning that a signal is issued if indicator A exceeds its threshold by $x\%$ and indicator B exceeds its threshold by $y\%$. To this end, for a pair of indicators (note there are 3 possible pairs overall), the authors use a grid search to find the x and y levels that maximize their objective function.

A.2.2 Inputs and Outputs

The authors use data from 18 industrial countries: Australia, Belgium, Canada, Denmark, Finland, France, Germany, Ireland, Italy, Japan, the Netherlands, New Zealand, Norway, Spain, Sweden, Switzerland, the United Kingdom, and the United States. For each country, annual data is used from 1970 to 2007. All trends are calculated using at least 10 years of data via a one-sided HP filter with $\lambda = 1,600$. Thus, the first year for which a signal can be issued is 1980. All the data inputs needed are provided by the BIS.

- **Input:** A country's annual equity gap is defined as the detrended total market capitalization (in real terms) of all of its listed companies.

- **Input:** A country's annual credit gap is defined as the detrended private-credit-to-GDP ratio.

- **Input:** A country's annual property gap is constructed by using both residential and commercial property prices, weighted with rough (according to the authors) estimates of their share in private sector wealth. This weighted property index is then detrended.

- **Input:** A list of crises for each country as defined in A.2.1.

- **Output:** For each pair of indicators, the optimal derived gap thresholds according to either the nts ratio or the true positive rate.

A.2.3 Empirical Findings

The authors find that the best performance is achieved if the credit gap exceeds 6% and either the equity gap exceeds 60% or the property gap exceeds a threshold that ranges from 15% to 25%. Especially for horizons of up to 2 years, a property-gap threshold of 15% is relatively attractive, as it predicts a high proportion of crises (some 70%), albeit it produces a higher percentage of false alarms. For a horizon of up to three years, a higher threshold is preferable, as financial distress does eventually emerge and the *nts* ratio is lower. As expected, the predictive power increases and the *nts* ratio decreases as the horizon is lengthened, confirming that the timing of the reversal of the financial imbalance is very hard to predict.

In terms of predicting the recent crisis in the U.S. (this is categorized as a crisis according to both definitions the authors use), the credit gap was above 6% since 2002, steadily increasing to a peak of 12.5% in 2007; furthermore, the property gap was above 15% since 2001 and reached a peak of 32% in 2005. Thus, the authors argue that the current crisis would have been predicted using the methodology described in this section.

A.3 Macroprudential Regulation

According to Borio (2010), a macroprudential framework involves calibrating supervision from a system-wide perspective, rather than from the individual institution basis. It also means taking explicitly into account the fact that drivers of risk depend endogenously on the collective behavior of financial institutions. The proximate objective of a macroprudential approach to regulation is to limit the risk of episodes of system-wide financial distress; its ultimate goal is to contain the costs they generate for the real economy, e.g., output loss.

For analytical convenience, it is helpful to think of the approach as having two dimensions. There is a time dimension, dealing with how aggregate risk in the financial system evolves over time. There is also a cross-sectional dimension, dealing with how risk is allocated within the financial system at a point in time. To each dimension corresponds a source of system-wide financial distress. In the time dimension, the source is the procyclicality of the financial system; in the cross-sectional dimension, the source is common exposures and interlinkages in the financial system. To address procyclicality, the principle is to build countercyclical buffers. To address common exposures and interlinkages, the principle is to calibrate prudential tools with respect to the contribution of each institution to systemic risk. This calibration can help ensure that each institution pays for the externality it imposes on the system.

The time and cross-sectional dimensions also differ on the measures used to assess their corresponding risks. In the time dimension, the ideal measure would be a fool-proof leading indicator of financial distress, with a lead sufficient to take remedial action. In the limit, one could envisage a framework analogous to inflation targeting: instruments would be adjusted so as to maintain the gauge within an acceptable range. In the cross-sectional dimension, the ideal measure would allow for a robust quantification of the contribution of each institution to systemic risk. However, given our state of knowledge, no single measure can perform the two functions simultaneously. In fact, measures that work well in the time dimension offer no guidance in the cross-section; and those that work in the cross-section can provide the wrong signals in the time dimension. If one tried to use these market-based measures

of systemic risk to address the time dimension, two problems would arise. The measures would provide the wrong signal: systemic risk would look low when, in fact, it was high. And adjustments for individual institutions' contributions to systemic risk would actually exacerbate procyclicality. This means that market-based measures, unless used as contrarian indicators of risk after suitable normalization, should not be part of leading indicators of aggregate risk.

With respect to implementing macroprudential regulatory policy, Borio (2010) argues for a bold approach in seeking to develop rules as far as possible. Especially in the time dimension, the political economy pressures and the temptation to discount risks can be too powerful. However, rules should be simple and understandable and a degree of discretion should be tolerated.

In Caruana's (2010b) speech on the macroprudential policies of Basel III given in October 2010 in China, he argues that if Basel III had existed prior to the crisis, banks would have faced it with much stronger capital bases, and would have been better able to draw on them. The financial system would have been much better prepared to withstand the shock of falling housing prices and losses on securitized assets. As a result, the negative feedback from losses to credit supply would have been milder, and governments would have had to provide less support. The aggravation of the business cycle (procyclicality) due to the financial system distress would have been significantly reduced.

One element of Basel III is the adoption of a countercyclical buffer for banks that is system-wide in its design. [35] This is based on the fact that private sector credit growth that is out of line with historical experience often ultimately imposes losses on the lenders. Thus the ratio of credit to GDP would serve as a common reference for the build-up phase of the buffer, which would be encouraged through restrictions on capital distributions.

Caruana (2010b) argues that in the U.S., since the early 2000s, the ratio of private credit to GDP and property prices both started going above their trends; a similar phenomenon happened in the U.K. and Spain during the same period. If the authorities in these three countries had responded to these observations in a manner consistent with the new counter-cyclical buffer, then the countercyclical capital requirement would have remained at more or less its maximum for some years and the authorities would have been well-advised to consider doing more. For example, loan-to-value ratios could have been lowered or tax deductions for mortgage interest rates on second homes could have been limited. In short, a countercyclical buffer at its maximum should be taken as a signal for further action. In all three economies, the countercyclical buffer could have been released some time after mid-2007 in response to financial strains and accumulating losses.

However, it should be noted that a late release of the countercyclical buffer would not have the same unwelcome consequences as a late requirement for it to be accumulated. If the authorities mandate the buffer too late, banks are left vulnerable to losses resulting from asset price declines; if the authorities release the buffer late, banks can still draw on it, but at the much lower cost of constrained distributions to shareholders.

---

[35]Under more recent iterations of Basel III—see Basel Committee on Banking Supervision (2011)—the countercyclical buffer is overshadowed by the "expected impact" approach.

B   Granular Foundations and Network Measures

Aggregate measurement of risks and imbalances does not capture everything. For example, aggregation typically tends to average away risk or dispersion in a data set. The financial sector and broader economy are complicated, noisy, and continuously evolving and simple aggregate metrics cannot transmit enough information to describe the full state of the system. To wit, the extant supervisory framework failed to prevent the 2007–09 crisis, despite an elaborate combination of measurement of aggregates, institution-level capital regulation and on-site examination. The systemic risk measures described in this section exploit a finer-grained instrumentation of the financial system and a more detailed understanding of the modalities of institutional stress and failure. In short, they explore more specifically *how* systemic events unfold.[36]

A discussion of some of the basic building blocks of financial risk measurement is a useful foundation for the subsequent discussion of specific approaches. Any measure of financial risk must involve payment obligations in some way.[37] Unless an event or episode provokes some form of default by participants on their promised cashflows, it is unlikely to create a systemic issue. Because of the central role of obligations, legal entities and the financial contracts between them are basic building blocks for understanding and modeling systemic risk.[38] This appears clearly in a network graph of the financial system, described in more detail below. By focusing attention on nodes (legal entities) and edges (contracts) the network graph emphasizes the role of contractual relationships, as well as a basic distinction between the individual legal entities (the focus of micro-prudential supervision) versus the financial system as a whole, which is the subject of macroprudential supervision. In this conception, each entity (i.e., obligor) is viewed as a "bundle of contracts" with other entities in the system, and measuring and modeling contracts becomes a central task for systemic supervision. Contracts define cashflow obligations, including contingent claims. Contracts are enforceable in court, so there are strong incentives for entities to be diligent in creating a new contract; there is reason to expect that each is reasonably complete and accurate declaration of commitments. One should anticipate that data used internally by financial firms for active business purposes will be more reliable than data created solely for regulatory reporting. Lastly, contracts serve as an atomistic input for most valuation and risk-management software packages; as a practical matter contracts are a basic unit of analysis.

Organizing risk measurement around the interacting concepts of contracts and entities creates an important requirement for two very basic types of identification data: legal entity identifiers (LEIs), and instrument types. We discuss this in more detail in Section 4.2, and

---

[36]A good example is Duffie (2010).

[37]Many financial contracts (or clauses within a contract) take the form of options or other contingent claims. Options confer a *right*, not an obligation, for one of the parties to the transaction. We use the term "obligation" here as shorthand that includes both traditional payment promises as well as option clauses and other contingent claims. If the option-holder exercises its right, the possibility of a payout commitment becomes an actual obligation. Similarly, for payouts that depend on an external contingency, these too become obligations when the identified contingency occurs.

[38]It is naive to expect that participants will never make payments unless they are legal obligated. This is not always the case, even when financial distress looms. For example, in Dec. 2007, Citigroup voluntarily brought onto its balance sheet $49B in "bankruptcy-remote" assets held in a structured investment vehicle (SIV). As a result of this unilateral move, Citigroup began making payments that had not previously been legal obligations of the firm.

assume for the remainder of this section that identification has been addressed, though we acknowledge that this is still a work in progress.

One key construct used for modeling granular interlinkages in the financial system is the counterparty network graph, described, for example, in International Monetary Fund (2009a). A graph is a mathematical formalism—popularized in social networking tools and degrees-of-separation games—consisting of a set of nodes (legal entities in this case) connected by a set of edges (contractual financial relationships). A fully detailed network model should make use of the legal entity and instrument type identifiers described above. Some of the natural applications of network tools include measuring the degree of connectivity of particular nodes to determine systemic importance, forecasting or simulating the likely contagion channels of institutional default or distress, and visualizing an overall "risk map" of exposure concentrations and imbalances in the system. For example, while an individual participant concerned about wrong-way risk exposures has only limited visibility into its guarantor's other commitments that could be diluting the guarantee, a systemic monitor with a view of the full counterparty network can see the accumulation of those other exposures. While a high-resolution (i.e., granular) examination of the graph at the level of individual contracts may be of interest, so too may be the larger imbalances revealed by an aggregation to the level of gross or net exposures between institutions, market sectors, or countries. Billio, Getmansky, Lo, and Pelizzon (2010) use observable stock market data to impute estimates of a latent network structure for four classes of institutions: banks, brokers, hedge funds, and insurance companies.[39]

The data requirements for a fully detailed counterparty network graph are potentially extensive. In addition to identifying each legal entity and contractual relationship, detailed attributes for each contract are also desirable because concentrated exposures may be contingent on future state and time through specific contractual terms and conditions. The status quo network graph might also be augmented with a conjectural set of possible new exposures, based on institutional rules (e.g., access to the discount window) or historical transaction data that expose pre-established counterparty relationships. All this additional detail should be helpful in observing concentrated exposures, or in tracing potential channels of crisis propagation through the system. Simulations of participants' portfolio adjustments as they ramify through the network from a systemic shock, such as the failure of a large dealer, can then reveal the likely systemic significance of such an event, as well as which entities are most vulnerable. On the other hand, several studies have attempted to infer the underlying interconnections in the system without directly observing them. For example, the default intensity model described in International Monetary Fund (2009a) focuses on direct and indirect systemic linkages among financial institutions. The default intensity model specifies a dynamic structure in which defaults occur at random, but tend to be clustered in time via an event intensity that both builds upon itself and is subject to abrupt random jumps. The funding gap model of Fender and McGuire (2010b) uses aggregate balance sheet data from the BIS on the banking-group level and the office-location level to infer the constraints faced by large banking organizations as they attempt to fund their local foreign

---

[39]Specifically, they construct a stock market index for each of the four sectors, and extract the four principal components for the returns series. As with the absorption ratio described below, the idea is that systemic risk will be indicated by a high concentration of the variance on the first principal component. They also examine various network measures of "directional causation" and Granger causality among returns for 100 large financial firms (25 chosen from each of their four sectors).

currency (especially USD) positions through their internal (i.e., within the banking group) and external networks.

In any financial interlinkage model, it is important to have the correct metric for the value of a contract or a portfolio. GAAP accounting provides the most familiar instance, whereby individual contracts are recorded on the general ledger at either historical cost or "fair value." Under GAAP, fair value is typically implemented as a mark-to-market valuation, or a mark-to-model when markets for a particular contractual position are illiquid or inactive.[40] Valuation technologies imply some fundamental requirements for risk measurement and monitoring, systemic and otherwise. Mark-to-market valuations require up-to-date prices on comparable securities (or replicating portfolios) traded in active markets. Mark-to-model valuations typically require projections of cashflows for futures dates and contingencies, which can be calculated from the detailed terms and conditions of the contract. In addition, marks to model require term structures of discount factors for converting those cashflows to present values. Note that valuation-based metrics are useful both at a systemic level as well as at the level of an individual firm; in particular, any measure implicitly or explicitly involving leverage is intrinsically valuation-based.

An important policy application of models of interconnectedness is the resolution plans now required for large financial firms. Section 165(d) of the Dodd Frank Act requires non-bank financial companies to report their ownership structure, assets, liabilities, contractual obligations, cross-guarantees, collateral pledges, major counterparties, and significant credit exposures. In essence, each reporting institutions is asked to provide the data to model its own node and connecting edges for the counterparty network graph. The goal is to facilitate an orderly resolution of the firm in the event of insolvency. In the event of the firm's insolvency, this information will make it possible to "cut" the connections that enmesh the failing firm with the financial network, thus insulating its counterparties from the knock-on effects that might otherwise propagate through the system. Equity-holders would be wiped out, and bondholders would get a "haircut" as conservators stepped in. Over time, an important balance to strike will be giving regulatory authorities sufficient information to implement their orderly resolution authority without making them responsible for the day-to-day operations of the firm.

The data requirements outlined above for orderly resolution authority are intense, but feasible (see the discussion in Section 4). In addition to the contractual details that establish the counterparty connections, analysis of the credit exposure report will require credit ratings, CDS spreads, and option prices to model the creditworthiness of the identified counterparties. To enable timeliness, this information set should be available—even if not collected—on a daily basis. To ensure operational capability in the heat of a crisis, a resolution plan might be subjected to occasional "fire drills" as a matter of course. Indeed,

---

[40]Sapra (2008) considers the trade-offs in the choice between these two valuation regimes, concluding that historical-cost treatment can lead to inefficient regulatory forbearance if asset values ignore current price signals, while mark-to-market treatment can lead to inefficient financial contagion if assets trade in illiquid markets with a propensity for significant short-lived price swings (especially in stressful episodes). In either regime, such scalar valuations have the important convenience that aggregation across positions is achieved as a simple sum, a fact that carries significant implications for a number of risk monitoring techniques, including Basel leverage calculations, asset-liability management (ALM), economic value of equity (EVE), etc. In a sense, a mark-to-market can be viewed as "crowd-sourcing" across many mark-to-model valuations, since the providers of market quotes typically arrive at their bids after some sort of modeling, even if unsophisticated.

if data formats could be sufficiently standardized to make data ingestion reliably seamless, regulators might even wait and collect the portfolio information from a troubled institution on an *ex post* schedule. This should allay some concerns of regulatory micro-management. Regulators have long had resolution authority, but have lacked the operational capability to resolve large, complex institutions in a timely way. The fall-back policy has essentially been one of "too big/complex to fail", which has the undesirable side effect of creating a moral hazard. Although a big job, the implementation of living wills has the potential to change that.

B.1 The Default Intensity Model

Giesecke and Kim (see International Monetary Fund (2009a)) propose a reduced-form statistical model for the timing of banking defaults which is designed to capture the effects of direct and indirect systemic linkages among financial institutions, as well as regime-dependent behavior of their default rates. The model is formulated in terms of a default rate, or "intensity", hence the term "default intensity model" (DIM). The default rate jumps at failure events, reflecting the increased likelihood of further events due to spillover effects. The magnitude of the jump is a function of the value of the default rate just before the event. The model specification guarantees that the impact of an event increases with the default rate prevailing at the event, a property that is supported by empirical observation. Indeed, the impact of an event tends to be "regime-dependent": it is often higher during a default clustering episode, when many firms are in a weak condition. However, the impact of an event dissipates over time.

B.1.1 Definition

Let T_n denote the sequence of economy-wide default times for the universe of Moody's-rated companies. Let N_t denote the number of defaults that have occurred by time t. Let λ_t denote the conditional default intensity, measured in defaults per year, and assume that it evolves through time according to the continuous time equation:

$$d\lambda_t \;=\; K_t(c_t - \lambda_t)dt \;+\; dJ_t \tag{A.7}$$

where $\lambda_0 > 0$ is the value of the intensity at the beginning of the sample period, $K_t = K\lambda_{T_{N_t}}$ is the decay rate with which the intensity reverts to the level $c_t = c\lambda_{T_{N_t}}$ at t, and J is a response jump process given by:

$$J_t \;=\; \sum_{n \geq 1} \max(\gamma, \delta\lambda_{T_n^-})I(T_n \leq t) \tag{A.8}$$

where $I(T_n \leq t) = 1$ if $T_n \leq t$ and 0 otherwise. The quantities $K > 0$, $c \in (0,1)$, $\delta > 0$, and $\gamma > 0$ are constant proportional factors, satisfying $c(1 + \delta) < 1$, to be estimated. Equation (A.7) states that the default rate jumps whenever there is a default, reflecting the increase in the likelihood of further events. This specification incorporates the impact of a default on the surviving firms, which is channeled through direct and indirect systemic linkages. The

magnitude of the jump depends on the intensity "just before" the event, which guarantees that the impact of an event increases with the default rate prevailing at the time of the event. The parameter γ governs the minimum impact of an event. After the intensity is ramped up at an event, it decays exponentially to the level $c\lambda_{T_{N_t}}$ with rate $K\lambda_{T_{N_t}}$. Since the reversion rate and level are proportional to the value of the intensity at the previous event, they depend on the times $T_1, \ldots, T_n$ and change adaptively at each default. For $T_n \leq t < T_{n+1}$, the behavior of the intensity is described by:

$$\lambda_t = c\lambda_{T_n} + (1-c)\lambda_{T_n}\exp\left(-K\lambda_{T_n}(t-T_n)\right) . \tag{A.9}$$

The vector parameter to estimate is denoted by $\theta = (K, c, \delta, \gamma, \lambda_0)$. The data consist of observations of economy-wide default times, T_n during the sample period $[0, \tau]$, which represents daily data from the time period January 1970 to December 2008. The maximum likelihood problem for the default rate $\lambda = \lambda^\theta$ is given by:

$$\max_{\theta \in \Theta} \int_0^\tau \left(\log\lambda_{s^-}^\theta dN_s - \lambda_{s^-}^\theta ds\right) \tag{A.10}$$

where Θ is the set of admissible parameter vectors. The optimization in (A.10) is done via the Nelder-Mead algorithm, which uses only function values. The algorithm is initialized at a set of random parameter values, which are drawn from a uniform distribution on the parameter space $\Theta = (0, 2) \times (0, 1) \times (0, 2)^2 \times (0, 40)$. The authors rerun the algorithm for 100 different initial parameter combinations to check the robustness of the estimates. An alternative brute force way of optimizing (A.10) is to perform a grid search over the 5 parameter values on the Θ space defined above.

The authors obtain the following estimates (we have included this as a guide to what "reasonable" numbers should look like): $\hat{\theta} = (0.11, 0.018, 0.13, 0.59, 32.56)$.

Having estimated the model for the economy-wide defaults, one can estimate the distribution of the number of economy-wide defaults during any future time period via a Monte Carlo simulation. Generating the future distribution of defaults is equivalent to generating a sequence of inter-arrival times which, in turn, can be generated sequentially by acceptance-rejection sampling from a dominating counting process with intensity $\lambda_{T_{N_{t^-}}} \geq \lambda_{t^-}$. Denote by τ the current time and H the end of the horizon of interest where $H > \tau$: thus, we seek to generate arrivals in $[\tau, H]$. The Monte Carlo steps are:

1. Initialize the time τ parameters $(N_\tau, T_{N_\tau}, \lambda_{T_{N_\tau}})$ according to the model in (A.9).
2. Set $n = N_\tau$, $S = \tau$, and $\lambda_S = c\lambda_{T_n} + (1-c)\lambda_{T_n}\exp\left(-K\lambda_{T_n}(S-T_n)\right)$.
3. **Begin loop**.
4. Draw $\epsilon \sim \exp(\lambda_S)$ and set $T = S + \epsilon$.
5. **If** $T > H$ then **exit loop**.
6. **Else, if** $T \leq H$: Set $\lambda_T = c\lambda_{T_n} + (\lambda_S - c\lambda_{T_n})\exp\left(-K\lambda_{T_n}(T-S)\right)$.
7. Draw $u \sim U(0, 1)$.
8. **If** $u \leq \lambda_T/\lambda_S$ then,
9.   Set $\lambda_T = \lambda_T + \max(\gamma, \delta\lambda_T)$,
10.   Set $T = T_{n+1}$ and update $n = n + 1$,

11. **End if**.
12. Set $S = T$ and $\lambda_S = \lambda_T$.
13. **End loop**.

One can run the above simulation multiple times and generate the distribution of economy-wide defaults over the period $[\tau, H]$. The last step in estimating the distribution of defaults in the financial sector is to assign each simulated default to a sector. A sector $s \in S = \{1, 2, \ldots, 12\}$ is selected with probability $\frac{Z(s)}{\sum_{s \in S} Z(s)}$, where:

$$Z(s) = \sum_{n=1}^{N_\tau} \frac{1}{1 + \tau - T_n} I(S_n = s) \tag{A.11}$$

where N_τ is the number of defaults observed during the sample period and $S_n \in S$ is the observed sector of the nth defaulter. More weight is assigned to recent observations, i.e., events that occur closer to the end of the sample period.

Given the estimated distribution of economy-wide events, the systemic risk measure is its 95% VaR, i.e., the number of defaults that would occur with a 5% probability, normalized by the number of firms at the beginning of the period. This measure of systemic risk is nice because it is not limited to financial sector defaults. Of course, one can capture the financial sector VaR specifics as well.

B.1.2 Inputs and Outputs

- **Input:** All corporate defaults from the period January 1, 1970 to December 31, 2008. The data were obtained from Moody's Defaults Risk Services. A default event is defined as either (1) a missed or delayed disbursement of interest or principal, including delayed payments made within a grace period; (2) bankruptcy (as defined by Chapters 7, 10, and 11 of the U.S. commercial code), administration, legal receivership, or other legal blocks to the timely payment of interest or principal; or (3) a distressed exchange that occurs when (i) the issuer offers debt-holders a new security or package of securities that amount to a diminished financial obligation; or (ii) the exchange had the apparent purpose of helping the borrower avoid default.

- **Output:** The default intensity model parameters $\theta = (k, c, \delta, \gamma, \lambda_0)$.

- **Output:** The distribution of future economy-wide and financial-sector defaults.

- **Output:** The 95% VaR of the distribution of economy-wide future defaults and the 95% VaR of the distribution of financial-system defaults as indicators of systemic risk.

B.1.3 Empirical Findings

The authors find that the DIM quite accurately captures the clustering of the economy-wide default events, thus suggesting the reliability of the model's out-of-sample forecasts. The authors also generate 1-year predictions of defaults over rolling quarters and find that the tail of the predicted 1-year distribution became quite fat during 2008, exceeding the levels

seen during the Internet bubble, suggesting a high probability of further banking failures. In particular, although the number of economy-wide failures for the whole episode of the Internet bubble-bust was substantially higher than the number of defaults observed during the recent crisis through the end of 2008, the 1-year predicted distribution of defaults as of the end 2008, i.e., the predicted number of defaults for 2009, had a much a fatter tail than that for the Internet episode, thus indicating the high likelihood of further defaults in 2009 and beyond.

When comparing the 95% VaR for banking sector defaults versus economy-wide defaults, during the 1998–2007 period, the banking sector proved more stable than the economy as a whole. However, during the 2007–2008 period, the authors find sharp parallel increases in the economy-wide VaR and the bank-wide VaR, which suggests a break with the past feedback patterns, indicating that macro-financial linkages are now tighter, potentially complicating the policy response to the financial sector problems.

The DIM was initially used in valuing CDOs and a detailed description can be found in Giesecke and Kim (2009).

B.2 Network Analysis and Systemic Financial Linkages

Chan-Lau, Espinosa, and Sole (2009) and the IMF's 2009 *Global Financial Stability Review* (International Monetary Fund, 2009a) contain two network models of interbank exposures to assess the network externalities of a bank failure using institutional data. Network analysis, which can track the reverberation of a credit event or liquidity squeeze throughout the system, can provide important measures of financial institutions' resilience to the domino effects triggered by financial distress. The starting point of any network analysis is the construction of a matrix of inter-institution exposures that includes gross exposures among financial institutions. Once an exposure matrix is in place, simulations can explore the effects of the individual default of each bank and then track the domino effects triggered by this event. For each bank, one can compute how vulnerable bank A is to bank B's default and which banks cause the most defaults.

Apart from being able to identify systemically important institutions as well as sensitive institutions, this approach can quantify the potential capital losses at various levels, from the bank level to the country level. Furthermore, the network approach allows for easy visualization of the risk map and contagion effects. The main shortcomings of this approach are that detailed inter-institution exposures are required (which is hard to get, especially for off-balance-sheet items) and that the modeling implicitly assumes static institution behavior.

B.3 Simulating a Credit Scenario

To assess the potential systemic implications of interbank linkages, a network of N institutions is considered. The analysis starts with the following stylized balance sheet identity for financial institution i:

$$\sum_j x_{ji} \; + \; a_i \;\; = \;\; k_i \; + \; b_i \; + \; d_i \; + \; \sum_j x_{ij} \qquad (A.12)$$

where x_{ji} stands for bank i loans to bank j, a_i stands for bank i's other assets, k_i stands for bank i's capital, b_i are long-term and short-term borrowing which are not interbank loans, x_{ij} stands for bank i borrowing from bank j, and d_i are the deposits in bank i.

To analyze the effects of a credit shock, the chapter simulates the individual default of each one of the N institutions in the network, and then tracks the domino effects resulting from each individual failure. More specifically, for different assumptions of loss given default (denoted by the parameter λ), it is assumed that bank i's capital absorbs the losses on impact, and the sequence of subsequent defaults triggered by this event are tracked. For instance, after taking into account the initial credit loss stemming from the default of institution h, the baseline balance sheet identity of bank i becomes:

$$\sum_j x_{ji} \; - \; \lambda x_{hi} \; + \; a_i \; = \; (k_i - \lambda x_{hi}) \; + \; b_i \; + \; d_i \; + \; \sum_j x_{ij} \qquad \text{(A.13)}$$

and bank i is said to fail when its capital is insufficient to fully cover its losses, i.e., when $k_i - \lambda x_{hi} < 0$. Subsequent "rounds" of the algorithm take into account the losses stemming from all failed institutions up to that point. Note that λ is typically assumed to be 1 because when the credit event first materializes, banks are unable to recover any of their loans as it takes time for secondary markets to price recently defaulted instruments. Simulations with $\lambda = 1$ should be interpreted as the "on impact" transmission of systemic instability.

B.4   Simulating a Credit-and-Funding-Shock Scenario

To analyze the effects of a credit-and-funding-shock scenario, it is assumed that institutions are unable to replace all the funding previously granted by the defaulted institutions, which, in turn, triggers a fire sale of assets. Thus, bank i is able to replace only a fraction $(1 - \rho)$ of the lost funding from bank h, and its assets trade at a discount, i.e., their market value is less than their book value, so that bank i is forced to sell assets worth $(1 + \delta)\rho x_{ih}$ in book value terms. Thus, the new balance sheet identity for bank i is given by:

$$\sum_j x_{ji} - (1+\delta)\rho x_{ih} - \lambda x_{hi} + a_i \; = \; (k_i - \delta\rho x_{ih} - \lambda x_{hi}) + b_i + d_i + \sum_j x_{ij} - \rho x_{ih} \; . \quad \text{(A.14)}$$

B.4.1   Inputs and Outputs

- **Input:** The authors use the cross-country bilateral exposures published in the *BIS International Banking Statistics* database for March 2008; they restrict themselves to the "Immediate Borrower Basis" part of the database, which consolidates the data by the residency of the immediate borrower. From this database, for each country's banking system, they obtain bilateral exposures for all non-bank asset values, all interbank asset values, all deposit liabilities, all non-bank debt obligations, and all interbank debt obligations. At the individual bank level, these data do not publicly exist although clearly, running such simulations at the individual bank level would be more useful.

- **Input:** Assumptions on the LGD of each bank (λ_i), the haircut in the fire sale of

assets (δ), and the liquidity parameter ρ. These assumptions are at the discretion of the regulator; in the paper, $\lambda = 1$, $\delta = 1$, and $\rho = 0.35$.

- **Output:** The number of induced failures due to an initial bank i failure.

- **Output:** The hazard rate for bank i, defined as the number of simulations in which bank i defaults (not including those in which bank i is the initial default bank) over the total number of simulations (not including those in which bank i is the initial default bank).

- **Output:** Total capital losses per bank.

B.4.2 Empirical Findings

The authors do not have access to individual bank data so they run the simulations on the country level by using the *BIS International Banking Statistics* data for March 2008. When simulating only credit events, they assume a λ of 1; they find that the U.S. and U.K. banking systems are by far the largest systemic risk contributors, as an initial default of these banking systems induces the greatest number of failures and the largest capital losses. Interestingly, they find that the most vulnerable banking systems are those of Belgium, the Netherlands, Sweden, and Switzerland as these countries have the highest hazard rates. For the case in which both credit and funding events are simulated, the authors find overall larger capital losses and higher hazard rates. France turns out to be an important liquidity provider as a French initial failure causes 3 defaults as opposed to 0 in the credit-event-only case. Furthermore, the systemic risk posed by a U.S. or U.K. default increases as well, given that these two countries' banking systems also provide liquidity.

The literature on using networks to model financial linkages is vast, and a good survey can be found in Upper (2007). Another paper which uses networks to study contagion risk in simulated banking systems is Nier, Yang, Yorulmazer, and Alentorn (2008). For some theoretical underpinnings of the network approach, see the seminal papers by Allen and Gale (2000) and Freixas, Parigi, and Rochet (2000). Because central banks typically have the data needed for such models, there are more references to individual-country banking systems, including most of the G10 countries.

B.5 Granger-Causality Networks

Billio, Getmansky, Lo, and Pelizzon (2010) propose two econometric measures of systemic risk that capture the interconnectedness among the monthly returns of hedge funds, banks, brokers, and insurance companies based on principal components analysis (PCA) and Granger-causality tests. The authors find that all four sectors have become highly interrelated over the past decade, increasing the level of systemic risk in the finance and insurance industries. Their measures can also identify and quantify financial crisis periods, and seem to contain predictive power for the current financial crisis. Their philosophy is that statistical relationships between returns can yield valuable indirect information about the build-up of systemic risk.

B.5.1 Principal Components Analysis

Increased commonality among the asset returns of banks, brokers, insurers, and hedge funds can be empirically detected by using PCA to decompose the covariance matrix of the four index returns. The returns for hedge funds is proxied by the CS/Tremont hedge fund database index, which is an asset weighted index of fund returns with a minimum of \$10 Million in AUM. The return for the "Broker" index is constructed by value weighting the monthly returns of all brokers in the CRSP database. The indexes for "Insurers" and "Banks" are constructed similarly. Given the monthly returns of these 4 indexes, the 4×4 covariance matrix can be estimated as:

$$\hat{\boldsymbol{\Sigma}} \;\equiv\; \frac{1}{T-4} \sum_{t=1}^{T} (\mathbf{R_t} - \bar{\mathbf{R}})(\mathbf{R_t} - \bar{\mathbf{R}})' \tag{A.15}$$

where T is the number of months used in the estimation and $\bar{R}$ denotes the vector of average returns. Given the covariance matrix, the 4 eigenvalues can be estimated along with the 4 eigenvectors. The authors use the above PCA analysis on 36-month rolling windows of returns and track the relative magnitudes of the four eigenvalues as well as the eigenvector exposures of the two eigenvectors corresponding to the two largest eigenvalues. The idea is that systemic risk is higher when the largest eigenvalue explains most of the variation of the data; and the "commonality" between the 4 types of institutions can be seen by the corresponding entries in the eigenvectors corresponding to the largest eigenvalues: if, for example, the eigenvector corresponding to the largest eigenvalue has similar entries, then all 4 types of institutions have similar exposure to this principal component.

B.5.2 Empirical Findings

The authors find that the first and second principal components capture the majority of return variation during the whole sample. Furthermore, the first principal component is very dynamic capturing from 65% to 93% of return variation; it seems to increase during times of stress. Interestingly, hedge funds seem to be quite independent of other financial institutions, with significant factor loadings on the third and fourth components, in contrast to the other 3 types of institutions which have significant loadings on the first two components. Therefore, hedge funds apparently do not contribute greatly to the covariance matrix of the four index returns.

B.5.3 Granger Causality

To investigate the dynamic propagation of systemic risk, the authors also measure the direction of the relationship between institutions using Granger causality: X is said to "Granger-cause" Y if past values of X contain information that helps predict Y above and beyond the information contained in past values of Y alone. The mathematical formulation is:

$$
\begin{aligned}
X_t &= \sum_{j=1}^{m} a_j X_{t-j} + \sum_{j=1}^{m} b_j Y_{t-j} + \epsilon_t \\
Y_t &= \sum_{j=1}^{m} c_j X_{t-j} + \sum_{j=1}^{m} d_j Y_{t-j} + \eta_t
\end{aligned}
\tag{A.16}
$$

where ϵ_t and η_t are two uncorrelated white noise processes and m is the maximum lag considered. The definition of causality implies that Y causes X when b_j is different from zero. Likewise X causes Y when c_j is different from zero. When both of these statements are true, there is a feedback relationship between the time series. The model selection criteria of the number of lags considered for the test is based on the Bayesian Information Criterion (see Schwarz (1978)). The causality is based on the F-test of the null hypothesis that coefficients b_j or c_j are equal to zero according to the direction of the Granger causality.

The authors analyze the pairwise Granger causalities between the t and $t + 1$ monthly returns of the 4 indexes; they say that X Granger-causes Y if c_1 has a p-value of less than 5%; similarly, they say that Y Granger-causes X if the p-value of b_1 is less than 5%. They adjust for autocorrelation and heteroskedasticity in computing the p-value.

The above Granger-causality analysis can be undertaken at the static level, i.e., over a given period, find the causal relationships or at the dynamic level, where the analysis is run at 36-month rolling windows and for each window, the dynamic causality index (DCI) is calculated as:

$$\text{DCI}_t \quad = \quad \frac{\text{number of causal relationships in window}}{\text{total possible number of causal relationships}} \qquad (A.17)$$

Clearly, an increase in the DCI indicates a higher level of system interconnectedness.

The authors take a step further and apply the dynamic Granger-causality methodology to individual institutions. For a given 36-month window, they select the 25 largest institutions from each of the four categories as determined by average market capitalization over the period for banks, insurers, and brokers, and by average AUM for hedge funds. They then compute the "directional" network of these 100 institutions using Granger causalities. The following set of risk measures can then be computed for each institution:

1. **Number of "In" Connections:** The number of financial institutions that significantly Granger-cause this financial institution.

2. **Number of "Out" Connections:** The number of financial institutions that are significantly Granger-caused by this financial institution.

3. **Number of "In+Out" Connections:** The sum of In and Out connections.

4. **Number of "In-from-Other" Connections:** The number of other types of financial institutions that significantly Granger-cause this financial institution. For example, for a hedge fund, "other types" are banks, brokers, and insurers.

5. **Number of "Out-to-Other" Connections:** The number of other types of financial institutions that are significantly Granger-caused by this financial institution.

6. **Number of "In+Out Other" Connections:** The sum of "In-from-Other" and "Out-to-Other" connections.

7. **Closeness:** The shortest path between a financial institution and all other financial institutions reachable from it, averaged across all other financial institutions.

8. **Eigenvector Centrality**: For a network with n nodes, let $\mathbf{A}$ be the adjacency matrix, the $(n \times n)$ matrix of 0's and 1's in which the (i, j) element is 1 if there is a connection between nodes i and j, and 0 otherwise. The eigenvector centrality measure is the eigenvector corresponding to the largest eigenvalue of $\mathbf{A}$.

9. **PCA:** The total absolute exposure of a financial institution to the first 20 principal components weighted by the percentage of the variance explained by each principal component.

An extension of the above Granger-causality approach is to apply a nonlinear version, which is presented in Billio and Di Sanzo (2006); this nonlinear approach takes into account the causality relationship based on both the means and volatilities of the financial institutions.

The authors present numerous results based on their Granger causality approach. First, they find that the DCI has been increasing since 1994, with a local peak in 1998, and another peak in 2007–2009; this is consistent with an increase in interconnectedness between institutions which presumably should have a relatively limited overlap in their lines of business.

In terms of evaluating their 9 risk measures presented above in predicting the recent financial crisis, the authors rank each institution for each risk measure from 1 to 100. They also compute the maximum percentage loss between July 2007 and December 2008 for each institution. Regressing the rankings of the maximum percentage losses for each institution on its rankings over the various systemic risk measures, after adjusting for leverage, they find that the most important systemic risk measures for an institution are PCA, "Out", "In+Out", "Out-to-Other", "In+Out Other", "Closeness", and "Eigenvector Centrality".

An aggregate systemic risk indicator for a given institution j over a given interval can thus be constructed by estimating the betas of the institution to the 9 risk factors and leverage, and then evaluating the sum:

$$
I_j \quad \equiv \sum_{i \in \text{risk factors}} \beta_i \left(\text{risk factor } i \text{ ranking of institution } j \right) . \tag{A.18}
$$

B.5.4 Inputs and Outputs

- **Input—Aggregate Index Case:** The monthly returns series for the industries of interest from January 1994 to December 2008. For hedge funds, this is the returns of the CS/Tremont hedge fund index. For brokers, the monthly returns and the market capitalizations of all companies with SIC Codes from 6200 to 6299 in the CRSP database are selected and the value weighted broker index is constructed. The indexes for banks and insurers are constructed similarly using SIC codes 6000 to 6199 for banks and 6300 to 6499 for insurers.

- **Input—Individual Institution Case:** The monthly returns series for the largest 25 institutions as measured by market capitalization over the 36-month period of interest. The authors run their analyses over rolling 36-month windows from January 1994 to December 2008. For brokers, insurers, and banks, the data are obtained from CRSP. For hedge funds, the TASS database is used, which reports individual hedge fund monthly returns and AUM.

- **Input—Individual Institution Case:** The monthly leverage of the institutions from January 1994 to December 2008, which is defined as total assets minus equity market value divided by equity market value. For banks, brokers, and insurers, this information is obtained from the merged CRSP/Compustat database. For hedge funds, they use reported average leverage from the TASS database.

- **Output—Both Cases:** The time series of the estimated covariance matrices using 36-month rolling windows, along with the time series of the corresponding eigenvalues and eigenvectors.

- **Output—Aggregate Index Case:** Granger causality relationships between the 4 industries for two periods: January 1994 to December 2000 and January 2001 to December 2008.

- **Output—Both Cases:** The DCI as defined in (A.17).

- **Output—Individual Institution Case:** The systemic risk indicator for each institution as defined in (A.18).

B.5.5   Related Studies and Further Reading

Chan, Getmansky, Haas, and Lo (2006b, 2006b) found that funding relationships between hedge funds and large banks that have brokerage divisions contribute to systemic risk. The interconnectedness of brokers and hedge funds has also been considered recently by King and Maier (2009), Aragon and Strahan (2009), Brunnermeier and Pedersen (2009), and Klaus and Rzepkowski (2009). Furthermore, Boyson, Stahel, and Stulz (2010) investigated contagion from lagged bank and broker returns to hedge fund returns.

B.6   Bank Funding Risk and Shock Transmission

Fender and McGuire (2010a) emphasize the fact that large, international banks tend to have offices in many different countries, hence focusing on banking-group-level balance-sheet risks may ignore important exposures at the individual-office level since these risks are netted out at the group level. Such individual-office frictions may be important, especially during a crisis. A key implication of geographic diversity is that cross-border linkages of individual office locations can determine how shocks are transmitted from one location (country) to another. The authors propose creating undirected and directed networks at the local-office level to assess these risks.

The authors also argue that group-level consolidated balance sheets may be misleading about funding risk. Only if resources available at one office location can immediately be used elsewhere, i.e., if banks' internal transfers of funds are perfectly frictionless, will group-level consolidated data provide an adequate picture of any vulnerabilities. As a result, when viewed from outside the bank using group-level (globally consolidated) data, stresses at the office location level can be masked, possibly generating a misleading picture of the overall degree of funding risk.

For example, at the height of the recent financial crisis, European banks faced problems borrowing dollars in their home and London offices. These offices typically fund purchases

of U.S.-dollar (USD) securities by borrowing USD or swapping domestic currency deposits into USD. In principle, their local offices, e.g., Korea, could simply send a part of their USD surplus to cover the needs of the London office (since, typically, local offices are net USD providers to the market because they borrow USD from their offices at home or elsewhere and swap them into local currency to invest in local assets). Yet, in practice, the extent to which this is possible depends on a variety of factors, including the nature of the local currency positions financed with these U.S. dollars, and whether these (and the FX swaps used to obtain local currency) can be unwound in a timely fashion. This unwinding can be particularly difficult in an environment where many banks are trying to do the same thing or are facing problems or restrictions in the relevant location. Thus, there is a need to be able to assess USD funding gaps not only at the aggregate bank-group level but also at the local-office level to develop a more complete understanding of all the possible vulnerabilities and their sources.

B.6.1 Definition

The authors use the *BIS International Banking Statistics* data, from which aggregate consolidated bank balance sheets can be constructed. However, the data are disaggregated in two important ways. First, for each national banking system (as defined by its headquarters location), the data provide a picture of the aggregated balance sheet of the underlying entities by office location (country level). Second, for each of these banking-system/office-location pairs, there is a partial breakdown of the location of the counterparties, for both assets and liabilities.

In terms of measuring a banking system's USD funding gap, the underlying off-balance-sheet FX swap positions must be inferred from reported on-balance sheet activities at the level of national banking systems. Specifically, assuming that banks have very small open FX positions, any on-balance-sheet net, i.e., assets minus liabilities, long or short position in a particular currency provides an estimate of the banks' offsetting net FX off-balance-sheet swaps positions in that currency. Gauging the funding risk arising from these activities requires information about the amount of banks' net short-term USD liabilities at any point in time. This, in turn, necessitates a breakdown by residual maturity of banks' USD-denominated assets and liabilities. Although maturity information is not available, the counterparty type (bank, non-bank or central bank) can serve as a proxy. The stylized balance sheet in Table A.3 shows the USD-denominated assets and liabilities of a banking system along with the assumption on their maturity (long-term versus short-term). Specifically, banks' USD claims on non-banks can be thought of as their desired USD-denominated investment portfolio. In contrast, interbank positions (both assets and liabilities) are typically short term, as are any FX swap positions used to convert funds into USD.

If liabilities to non-banks are all assumed to be long-term, then the lower bound estimate of these banking systems' overall USD funding gap is:

$$\text{USD Funding Gap Lower Bound} = \text{net USD interbank borrowing} + \qquad\text{(A.19)}$$
$$\text{net USD borrowing from monetary authorities} +$$
$$\text{net USD borrowing from the FX swap market}$$
$$= \text{net USD claims on non-banks} -$$
$$\text{net USD interbank liabilities}$$

The upper-bound estimate is then set by simply adding liabilities to non-banks to the lower bound measure above; in other words, it is equal to:

$$\text{USD Funding Gap Upper Bound} = \text{Net USD claims on non-banks} \qquad\text{(A.20)}$$

Assets (in USD)	Liabilities (in USD)
Claims on non-banks (assumed long-term)	Liabilities to non-banks (short- or long-term)
	Net interbank borrowing (short-term)
	Net borrowing from monetary authorities (short-term)
	Net borrowing via the FX swap market (short-term)

Table A.3: Stylized USD-denominated balance sheet.

The authors run the funding gap analysis above on two levels: the group level and the office-location level. The group-level funding gap estimates are constructed by aggregating banks' global balance sheets into a consolidated whole, and then calculating funding risk on this aggregated balance sheet. The office-location level estimates are constructed by calculating funding risk at the office-location level, and then aggregating the series up across office locations for each banking system. By construction, the office-level estimates should be at least as large as the corresponding group level estimates.

In terms of assessing the country-to-country bank linkages, the authors use the *BIS Locational Banking Statistics by Residency* to look at interlinkages at the country level. These statistics include the size, currency, counterparty type and, critically, the counterparty location of claims and liabilities of banks in one country to borrowers located in another country. They do not contain information on the nationality of the reporting banks in each location. Thus, the data provide a particular picture of geographical, i.e., country-to-country, interlinkages and the flow of funds between them, but are less suited for more structural balance sheet analysis.

In creating an undirected network, the size of each node is proportional to the stock of cross-border claims and liabilities of reporting banks located in the particular geographical region. The thickness of a line between regions A and B is proportional to the sum of claims of banks in A on all residents of B, liabilities of banks in A to non-banks in B, claims of banks in B on all residents of A, and liabilities of banks in B to non-banks in A. Note that this undirected network is a static depiction of cross-border linkages.

In creating a directed network, times t_1 and t_2 must be defined. The thickness of an arrow is then proportional to the amount of net bank flows between regions from t_1 to t_2. More specifically, an arrow points from A to B if net flows in this direction are positive, calculated as changes in net interbank claims (assets minus liabilities) of banks in A on banks in B, plus net claims of banks in A on non-banks in B, minus net claims of banks in B on non-banks in A.

B.6.2   Inputs and Outputs

In measuring funding risk, ideally one would have data that provide a geographically disaggregated picture of banks' consolidated balance sheets. That is, data in which the structure of banks' global operations (for both assets and liabilities) is visible, and which contain some level of information on banks' operations in various locations, and the interlinkages between these local offices, i.e., inter-office positions, and non-affiliated entities. Currently no dataset exists with this level of detail, nor is such data likely to be available any time soon. Thus, the authors use the *BIS International Banking Statistics*.

- **Input:** The group-level balance sheets for each national banking system (as defined by banks' headquarters location): these are obtained from the *BIS Locational Banking Statistics by Nationality* database. The authors assess quarterly group-level USD funding risk estimates over the period from 2000:Q1 to 2010:Q1.

- **Input:** The balance sheet of the underlying entities by office location (country level) including counterparty locations which are calculated from the *BIS Locational Statistics by Residency* data. For the office-location level USD funding risk estimates, the authors use quarterly data from 2000:Q1 to 2010:Q1. For the undirected network, the authors use the data from 2010:Q1; for the directed network, the authors set $t_1 = 2000$:Q1 and $t_2 = 2007$:Q2.

- **Output:** USD funding risks at the group-level and at the office-location level.

- **Output:** Directed and undirected networks of international banking system linkages at the office-location level.

B.6.3   Empirical Findings

In evaluating the USD funding risks, the authors find that their measures are rough, with very wide ranges between bounds. However, the indicators do seem to confirm that funding risks are actually larger than consolidated data would make them appear. This is a result of the netting of interbank and FX swap positions in the group-level estimates. These effects can be rather large, as suggested by the differences in the lower-bound indicators between the group-level and office-level cases for French, Dutch and Belgian banks. Moreover, analysis of the underlying office-location-level funding risk measures indicates that a significant portion of the total dollar funding risk is attributable to a given banking system's foreign offices, about which home country regulators may have only limited information.

In terms of assessing systemic linkages in the international banking system, the authors find that capital flows saw a phenomenal reversal in the wake of the recent crisis, in particular

out of the United States. Until mid-2007, banks facilitated international capital flows out of Japan and the Eurozone area as well as from Asian financial centers and oil-exporting countries. Banks routed these funds via offices in the United Kingdom and in Caribbean financial centers, ultimately transferring them to borrowers in the United States and in emerging markets. After the start of the crisis, the direction of many of the bilateral flows reversed, in part generated by capital movements back to the U.K.

A more detailed exposition of estimating USD funding gaps can be found in Fender and McGuire (2010b) and Lee (2010).

B.7 Mark-to-Market Accounting and Liquidity Pricing

The most prominent systemic-risk issue associated with mark-to-market accounting is the potential for fire-sale contagion, as forced sales can depress market prices provoking additional deleveraging. Laux and Leuz (2010) survey the issues. Adrian and Shin (2010) provide empirical evidence of procyclicality in leverage. They also find that increases in collateralized lending in the repo market has significant forecasting power for market volatility. Sapra (2008) considers the trade-offs between historical-cost accounting and mark-to-market accounting in the context of a stylized 2-period model in which the banking and insurance sectors interact with liquidity pricing to generate financial contagion.

Proponents of mark-to-market accounting argue that the market price of an asset or liability leads to better insights into the risk profile of firms than a historical-cost-based measurement system. However, moving from a historical-cost regime to a mark-to-market regime without addressing the other imperfections in the financial system does not guarantee a welfare improvement because mark-to-market accounting creates contagion between the banking and insurance sectors whereas under historical-cost accounting, there is no contagion. The key friction that leads to contagion in the financial system is liquidity pricing; in times of liquidity shortage, the interaction among institutions and markets can lead to situations where prices no longer reflect fundamentals but rather the amount of liquidity available to buyers in the market. For both banks and insurers, a large proportion of their balance sheets consists of illiquid claims; these claims are not standardized and do not trade in deep and liquid markets. The historical-cost regime relies on past transaction prices resulting in accounting values that are insensitive to current price signals. This lack of sensitivity to price signals induces inefficient sales because the measurement regime does not reflect the appreciated value of the measured assets. Marking to market overcomes this price insensitivity by extracting the information conveyed by current price signals, but in trying to extract information about market prices, marking to market potentially adds endogenous volatility to prices.

The main result of the author's stylized model is that under historical-cost accounting, even though fundamentals may be sour, the banks' solvency requirements rely on past prices and therefore may incorrectly allow banks to continue operating. By the time regulators intervene, it is too late because the size of the banks' assets is likely to have declined considerably and everyone is worse off. Under mark-to-market accounting, the solvency requirements rely on current price signals and are therefore informative about the value of the banks' assets, allowing regulators to efficiently shut down the banks earlier, before losses escalate. However, in the presence of liquidity pricing, contagion is a necessary evil.

The implication of this trade-off is that mark-to-market accounting would dominate

historical-cost accounting if the welfare losses from contagion are relatively small compared to the welfare losses from inefficient continuation under historical-cost accounting. All else being equal, the more illiquid the assets of the bank and therefore the more severe the liquidity pricing effect, the more likely it is that historical-cost accounting would dominate mark-to-market accounting from a welfare standpoint.

C   Forward-Looking Risk Measurement

A natural extension of valuation-based measures is the broader set of forward-looking risk measures. Such metrics can be scalar-valued, like value at risk (VaR) or a CDS spread, or multi-dimensional, like the coefficients of a multi-factor capital asset pricing model (CAPM). This broader class of risk metrics expands the set in at least three ways. First, when contrasted with historical-cost valuation, a forward-looking measure should aid in understanding the evolution of a portfolio, institution, or financial system, even if the metric is not intended as a forecasting tool. A stark example of the problem is a forward contract, which involves no up-front payment, but which nonetheless exposes the counterparties to significant risk. Second, unlike valuations, aggregation across positions or portfolios is not necessarily linear, so that simply summing the statistics is typically inappropriate.[41] For example, the VaR of a portfolio is not the (weighted) sum of the VaRs of its individual component securities; it depends importantly on correlations and other higher-order attributes of the multivariate returns distribution. A traditional balance sheet cannot capture such relationships. Third, non-linearity may complicate measurement: unlike simple point estimates, metrics such as convexity or gamma may explore higher-order contours of a valuation function; similarly, tail dependence statistics and volatility clustering parameters are commonly used to describe higher-order aspects of the returns distribution.

In terms of systemic risk monitoring, assessing forward risk measures may also be useful in stress testing financial institutions: ideally, some of the stress scenarios should depend on the forward looking risk metrics.

For purposes of risk measurement—systemic or otherwise—it is important to have a forward-looking view of the cashflows for positions and portfolios at different times in the future, and under varying circumstances. As a policy matter, such forward-looking metrics may help focus regulatory scrutiny on emerging risk factors and other exposures before they begin to appear in traditional financial statements. Because the future is not precisely knowable, risk modeling often postulates one or more probability distributions or stochastic processes to capture systematically the patterns that describe the evolution of a portfolio or of broader aspects of the financial system. As a familiar example, a short-horizon VaR statistic might model returns as independent draws from a normal probability distribution, with more sophisticated versions incorporating mean-reversion or volatility clustering. Gray and Jobst (2010) apply Merton (1973)'s model in which equity is viewed as a call option on the assets of the firm (exercisable by paying the face value of the firm's debt at maturity), and the value of total assets evolves according to a geometric Brownian motion. Among other things, the model generates estimates of the implicit probability of failure/default using observed equity prices as inputs. They apply CDS spreads to back out the proportion of the expected

---

[41]This is not always the case. For example, the marginal risk contributions defined in Huang, Zhou, and Zhu (2009a) retain the additivity property.

loss absorbed by the private sector, with the expected residual loss imposed on the taxpayer as a systemic impact. The financial turbulence model of Kritzman and Li (2010) considers a multivariate case, such as returns on securities in a portfolio. They define "turbulence" as divergence from typical behavior, and measure it as the Mahalanobis distance of a particular returns vector from what one should expect based on past observation.[42]

The measures of Capuano (2008) and Segoviano and Goodhart (2009) both apply maximum-entropy techniques to infer a probability distribution for one or more variables of interest.[43] Capuano (2008)'s Option Implied Probability of Default (iPoD) model uses maximum entropy to extract market-based default probabilities from prices of equity options. The process exploits the entire information set available from option prices, including volatility smile and skew. One can also obtain the implied expected value of equity, leverage, and the Greeks.[44] Segoviano and Goodhart (2009) model systemic risk via the so-called banking system's multivariate density (BSMD) function, where they define the banking system as a portfolio of banks. The maximum-entropy estimation is constrained by the "observed" probabilities of distress (PoDs) for the individual institutions, measured via several alternative techniques, including equity options and spreads on credit default swaps. They then use the estimated BSMD to produce several systemic stability metrics: joint probability of default (JPoD); banking stability index (BSI); distress dependence matrix (DDM); and probability of cascade effects (PCE). These measures embed the interdependence structure of distress across banks, capturing not only linear correlation but also nonlinear distress dependencies. Moreover, the structure of dependencies changes endogenously as banks' PoDs change, so that the stability measures remain consistent with the economic cycle.

An important class of structural models connects the performance of financial portfolios, such as the cashflow behavior of the underlying financial contracts, to some set of external factors. In some cases, such as the underlying for a derivative security, these factors are explicitly specified, including the rule(s) determining cashflows as a function of the factors. In contrast, sometimes the particular factors involved and their relationship to the performance of the portfolio is less explicit. For example, in the housing sector model of Khandani, Lo, and Merton (2009) described above, changes in housing prices have a causal impact on lenders' portfolio losses, even though this empirical relationship is not fixed by a legal commitment. As discussed below, many stress-test approaches have this same flavor.

A scorecard analysis of consumer creditworthiness is another common application of

---

[42]Intuitively, the measure assumes a series of concentric, similar (same shape, different sizes) ellipsoidal shells that enclose progressively larger proportions of the scatter plot of historical observations, where the shape of the shells is derived from the covariance matrix of the observations. The Mahalanobis measure is essentially the size of the smallest ellipsoidal shell that just encloses the point (i.e., returns vector) in question.

[43]More specifically, they use the principle of minimum cross-entropy, described in Cover and Thomas (2006). In a nutshell, the maximum entropy method chooses the "best" distribution from among all probability distributions consistent with the known facts about the data-generating process, where "best" is defined as the distribution with the most entropy (or least explanatory power). In other words, it chooses the distribution that embeds the fewest extraneous assumptions about the data, beyond what is directly observable.

[44]The "Greeks" is a slang term in mathematical finance for various commonly used model parameters and risk sensitivity measures for derivatives contracts. The name derives from the fact that these variables are typically represented by letters of the Greek alphabet (including a made-up letter, "vega", that does not exist in the Greek language).

an external-factors approach. For example, Khandani, Kim, and Lo (2010) apply machine-learning to transactions and credit bureau data from January 2005 to April 2009 for a sample of customers of a single bank. They construct nonlinear, non-parametric, out-of-sample forecasts of consumer credit risk that significantly improve the classification rates of credit-card delinquencies and defaults.[45] The forecast proportion of delinquencies in the population for a given horizon becomes an indicator of systemic risk emanating from consumer lending.

The absorption ratio (AR) of Kritzman, Li, Page, and Rigobon (2010) posits a latent linear factor structure—essentially, a black box of unnamed external factors—to model the driving forces underlying the evolution over time of a multidimensional system. The AR is proportion of the variance in the system explained or "absorbed" by a fixed number of factors.[46] A higher AR indicates that markets are more tightly coupled, suggesting that shocks will propagate through the system more quickly. An increased AR is forward-looking inasmuch as it points to heightened systemic fragility, although by itself this need not lead to volatility or distress. Empirically, they find that the AR for the industry sectors in the MSCI USA index is highly negatively correlated with stock prices and that all of the most significant U.S. stock market drawdowns followed AR spikes within a month. When using the 14 subindexes of the Case-Shiller index for housing prices, they find that the AR was a leading indicator of the U.S. housing market bubble.

The data requirements for forward-looking risk metrics and stress testing are extensive. Models that rely on currently and publicly available data—e.g., Capuano (2008), Segoviano and Goodhart (2009), Adrian and Brunnermeier (2010), International Monetary Fund (2009a), Huang, Zhou, and Zhu (2009b), Kritzman, Li, Page, and Rigobon (2010)—naturally leverage the forward-looking information value of market prices. This includes equity and option prices and CDS spreads for financial institutions, but also markets with broader scope, such as stock market price and volatility indexes, exchange rates and housing and commodities prices. Models that posit a latent factor structure—e.g., Khandani, Lo, and Merton (2009), Adrian and Brunnermeier (2010), International Monetary Fund (2009a), Huang, Zhou, and Zhu (2009b), Kritzman, Li, Page, and Rigobon (2010)—tend to focus especially on the broader indicators such as interest rates, house-price indexes, aggregate corporate bond spreads, and equity and volatility indexes to summarize the systemic drivers. An exception is Khandani, Kim, and Lo (2010), who had access to confidential portfolio data for a single institution. Stress testing approaches such as Hirtle, Schuermann, and Stiroh (2009) or Duffie (2011) typically require very specific information about participants' portfolios, including contractual terms and conditions and the details of risk-model configurations. This allows them to map out in detail the contingent cashflows under various scenarios; because the information needs are so intensive, the former occurred only under a regulatory mandate, while the latter remains a proposal. Going forward, the OFR will have the authority

---

[45]Specifically, they use generalized classification and regression trees (CARTs) to construct their forecasts. In a CART classifier, the "output" variable (continuous or discrete) is related to a set of independent or "input" variables through a recursive sequence of simple binary relations. CART models are well suited to problems with high dimensional feature spaces. The forecasted delinquencies are highly correlated with realized delinquencies. The linear regression R^2 is 85% for both 6- and 12-month horizons, indicating that the forecasts are capturing the dynamics of the consumer credit cycle over the sample. However, the raw forecasts consistently underestimate the absolute level of delinquencies by a scalar multiple, a bias that is corrected by training the estimator on its Type I and Type II errors.

[46]The latent factors and their loadings are extracted via principal components analysis.

to assemble this information. Because detailed portfolio data involve business confidential information, adequate security precautions will be essential.

C.1   Contingent Claims Analysis

Gray and Jobst (2010) propose using contingent claims analysis (CCA) to measure systemic risk from market-implied expected losses, with immediate practical applications to the analysis of implicit government contingent liabilities, i.e., guarantees. In addition, the framework also helps quantify the individual contributions of financial institutions to overall contingent liabilities in the event of a systemic distress. Based on a sample of the 36 largest financial institutions (banks, insurance companies, and asset managers), this systemic risk measurement framework generates an estimate of the joint contingent liabilities from market-implied government support. This approach does not only quantify the magnitude of potential risk transfer to the government but also helps indicate the contribution of individual institutions to contingent liabilities over time.

C.1.1   Definition

The CCA approach is based on the work pioneered by Merton (1973), which posits that the equity of a firm can be viewed as a call option on its assets and the debt of the firm can be modeled as being long risk-free debt and short a put option on the firm's assets. CCA determines the risk-adjusted balance sheet of firms, based on the balance sheet identity that at each instant, a firm's assets (A_t) should be equal to the sum of the market value of its oustanding debt (D_t) and its outstanding equity (E_t). Mathematically:

$$A_t \;=\; D_t \;+\; E_t \; . \tag{A.21}$$

Note that the market value of outstanding debt D_t is different than the face value of outstanding debt B which is due at time T. In order to be able to value the embedded put option in the firm's price of debt, the asset value and volatility need to be estimated. Note that the asset value here is not the book value of the assets; it is the "market value" of the assets and is not directly observable. However, the market value of the equity (E_t) and its volatility (σ_E) are observable. Thus, given an asset pricing model, one can back out the current level of the asset value and its volatility. The authors use the standard Brownian drift-diffusion model used in the Black-Scholes-Merton (BSM) option pricing model (see Black and Scholes (1973)):

$$\frac{dA_t}{A_t} \;=\; rdt \;+\; \sigma_A dZ_t \tag{A.22}$$

where r is the (risk-neutral) drift of A_t, σ_A is its volatility, and Z_t is a standard geometric Brownian motion. Note that under the risk-neutral measure, the drift is the same as the risk free rate. Given the above asset price model, the price of the equity, which is modeled as a European call option on the assets with a strike price of B, is given by (according to

the BSM model):

$$E_t \;=\; A_t\phi(d_1) \;-\; B\exp(-r(T-t))\phi(d_2) \tag{A.23}$$

$$d_1 \;=\; \frac{\ln(A_t/B) + (r + \sigma_A^2/2)(T-t)}{\sigma_A\sqrt{T-t}}$$

$$d_2 \;=\; d_1 \;-\; \sigma_A\sqrt{T-t}$$

where $\phi(\cdot)$ denotes the cumulative distribution function (CDF) of the standard normal density function. Furthermore, the following relationship holds:

$$E_t\sigma_E \;=\; A_t\sigma_A\phi(d_1) \;. \tag{A.24}$$

Thus, one can solve two equations (A.23) and (A.24) for two unknowns (A_t and σ_A). Given these parameters, the value of the risky debt can be evaluated: it is equal to the default-free debt minus the present value of the expected losses due to default, i.e., the price of a put option on the firm's assets:

$$D_t \;=\; B\exp(-r(t-t)) \;-\; P_E(t) \;. \tag{A.25}$$

The price of the put option can be computed as:

$$P_E(t) \;=\; B\exp(-r(T-t))\phi(-d_2) - A_t\phi(-d_1) \;. \tag{A.26}$$

The implicit put option calculated for each financial institution from equity market and balance sheet information using CCA can be combined with information from CDS markets to estimate the government's contingent liabilities. If guarantees do not affect equity values in a major way, CDS spreads should capture only the expected loss retained by the financial institution after accounting for the government's implicit guarantee. Hence, the scope of the market-implied government guarantee is defined as the difference between the total expected loss, i.e., the value of a put option $P_E(t)$ derived from the firm's equity price, and the value of an implicit put option derived from the firm's CDS spread. The price of the "CDS put option" is:

$$P_{CDS}(t) \;=\; \left(1 - \exp\left(-(s_{CDS}(t)/10000)(B/D(t) - 1)(T-t)\right)\right)Be^{-r(T-t)} \;. \tag{A.27}$$

Given the CDS put option, we can evaluate the fraction:

$$\alpha(t) \;=\; 1 \;-\; P_{CDS}(t)/P_E(t) \tag{A.28}$$

of total potential loss due to default covered by implicit guarantees that depress the CDS spread below the level that would otherwise be warranted for the option-implied default risk. In other words, $\alpha(t)P_E(t)$ is the fraction of default risk covered by the government and $(1-\alpha(t))P_E(t)$ is the risk retained by an institution and reflected in the CDS spreads. Thus, the time pattern of the government's contingent liability and the retained risk in the financial sector can be measured.

The measure of systemic risk is given by summing the guarantees over all n institutions in the sample:

$$\sum_{i=1}^{n} \alpha_i(t)P_E^i(t) \ . \tag{A.29}$$

The authors also talk about an extension called "systemic CCA" which involves applying the concept of extreme value theory (EVT) in order to specify a multivariate limiting distribution that formally captures the potential of extreme realizations of contingent liabilities. However, they do not present the methodology in Gray and Jobst (2010) and the paper they reference (by the same authors) has not yet been published (see Gray and Jobst (2011)).

C.1.2 Inputs and Outputs

The authors ran their analysis for a set of large commercial banks, investment banks, insurance companies, and special purpose financial institutions between January 1, 2007 and January 31, 2010, but they do not specify which ones.

- **Input:** Daily at-the-money equity options data from Bloomberg.

- **Input:** The default horizon T, which is set to 1 year.

- **Input:** The default barrier B, which is set equal to total short-term debt plus one-half of long-term debt, consistent with the approach taken by Moody's KMV CreditEdge. This is re-estimated each quarter for each firm based on its quarterly financial accounts.

- **Input:** The risk-free rate of interest of 3%.

- **Input:** Daily 1-year CDS spreads obtained from Markit.

- **Output:** $P_E(t)$ as defined in (A.26).

- **Output:** The systemic risk indicator as defined in equation (A.29).

C.1.3 Empirical Findings

The authors find a secular increase of implied financial sector support a result of the widening gap between default risk implied by equity and CDS put option values. Sudden declines in alpha-values around April 2008, October 2008, and May 2009 are attributable to a greater alignment of changes in market capitalization and the market price of default risk, as explicit

or implicit financial sector support have little or no impact on slowing the rise of CDS spreads amid falling equity prices of distressed firms.

Furthermore, the authors find that total expected losses, i.e., the sum of $P_E(t)$ over all institutions, are highest between the periods just after Lehman's collapse (September 2008) and end-July 2009; the peak is at 1% of GDP in March 2009. At the peak of the crisis, more than 50% of total expected losses could have been transferred to the government in the event of default (0.5% of GDP).

C.2 Mahalanobis Distance

Kritzman and Li (2010) define "financial turbulence" as a condition in which asset prices, given their historical patterns of behavior, behave in an uncharacteristic fashion, including extreme price moves, decoupling of correlated assets, and convergence of uncorrelated assets. They quantify turbulence via the Mahalanobis distance (see Merton (1937)), which measures the statistical unusualness of a set of returns given their historical pattern of behavior. Their measure is very general and can be applied across asset classes for which time-series return data are available.

C.2.1 Definition

Given the returns of n assets, one can define the financial turbulence measure as the squared Mahalanobis distance, as:

$$d_t \;=\; \left(\mathbf{y_t} - \mathbf{m}\right)' \mathbf{\Sigma}^{-1} \left(\mathbf{y_t} - \mathbf{m}\right) \tag{A.30}$$

where:

$$
\begin{aligned}
d_t &= \text{turbulence at time } t \\
\mathbf{y_t} &= (n \times 1) \text{ vector of asset returns} \\
\mathbf{m} &= (n \times 1) \text{ sample average vector of asset returns} \\
\mathbf{\Sigma} &= (n \times n) \text{ sample covariance matrix of asset returns}
\end{aligned}
$$

By running this metric over time, one can generate the path of financial turbulence over time and define as "turbulent" those days for which d_t is above its 75th percentile (the rest are referred to as "quiet" days).

The application of this systemic risk measure can be used in stress testing portfolios of assets. In estimating a portfolio's VaR, the authors suggest using data only from the turbulent periods instead of the full sample. Thus, this turbulence-adjusted VaR better reflects asset correlations and returns during a turbulent state and is, therefore, a more realistic estimate of possible losses arising from a systemic event.

C.2.2 Inputs and Outputs

- **Input:** Monthly returns from January 1973 to December 2009 for U.S. Stocks (S&P 500), non U.S. Stocks (MSCI non-U.S. index), U.S. Bonds, Real Estate, and Commodities (they do not specify the data source for the last 3). In general, the frequency and asset selection can be tailored according to the application.

- **Output:** The financial turbulence measure in equation (A.30).

- **Output:** Turbulence-adjusted VaR for a given portfolio.

C.2.3 Empirical Findings

Two empirical features of turbulence are particularly interesting. First, the authors find that returns to risk are substantially lower during turbulent periods than during non-turbulent periods, irrespective of the source of turbulence. Moreover, they find that financial turbulence is highly persistent. Although the initial onset of financial turbulence cannot be predicted, once it begins, it usually continues for a period of weeks as the markets digest it and react to the events causing the turbulence.

The authors also test portfolios containing various combinations of U.S. and international stocks, U.S. and international bonds, commodities, and real estate. They find that the turbulence-adjusted VaR does a better job of forecasting maximum portfolio losses during the financial crisis compared to the standard VaR statistic, which severely underestimated the riskiness of these portfolios. To be clear, the turbulence-based approach does not offer a more reliable estimate of when an extreme event will occur; rather, it gives a more reliable estimate of the consequences of such an event.

Other measures of financial distress have been pursued in the literature as well. Correlations conditioned on upside or downside market moves are presented in Ang and Chen (2002); time-varying volatility models, such as GARCH models (see for example Bollerslev (1986)); Markov regime-switching models (see Ang and Bekaert (2002)); and implied volatility (see Mayhew (1995)) have all been proposed as measures of financial distress.

C.3 The Option iPoD

Capuano (2008) proposes an Option Implied Probability of Default (iPoD) in which market-based default probabilities are inferred from equity options by applying the principle of minimum cross-entropy (see Cover and Thomas (2006)). The principle of maximum entropy and the related minimum cross-entropy principle make it possible to recover the probability distribution of a random variable. Thus, there is no distributional assumption placed on the asset nor is there a need to specify any assumption on recovery rates. Furthermore, the default barrier is endogenously determined.

The framework exploits the entire information set available from option prices, so as to capture the well documented volatility smile and skew. One can also obtain the implied expected value of equity and leverage as well as the "Greeks".

C.3.1 Definition

The probability of default (PoD) is defined as:

$$\text{PoD}(X) \quad = \quad \int_0^X f_v dv \tag{A.31}$$

where f_v is the probability density function (PDF) of the value of the asset, and X is the default threshold. Solving for the PoD uses the cross-entropy functional introduced by Kullback and Leibler (see Kullback and Leibler (1951)). The recovered distribution is solely driven by what the researcher can observe. There is no additional assumption. In this sense, the maximum entropy distribution is the closest to the true distribution, as long as the true distribution is reflected in the observable data.

The problem to be solved is:

$$\min_D \left\{ \min_{f(V_T)} \int_{V_T=0}^{\infty} f(V_T) \log \left[\frac{f(V_T)}{f^0(V_T)} \right] dV_T \right\} \tag{A.32}$$

where $f^0(V_T)$ is the prior probability density function of the value of the asset at time T, representing the researcher's prior knowledge on $f(V_T)$, the posterior density. The integrand is the cross-entropy between the prior and the posterior; it represents the degree of uncertainty around the posterior. The minimization problem posed in (A.32) is subject to the following 3 constraints:

- **Balance Sheet Constraint:** The equity can be viewed as a call option on the assets:

$$E_0 \quad = \quad e^{-rT} \int_{V_T=D}^{\infty} (V_T - D) f(V_T) dV_T \ . \tag{A.33}$$

- **Observable Options Pricing Constraint(s):** The resulting density should be able to price observable option prices:

$$C_0^i \quad = \quad e^{-rT} \int_{V_T \ = \ D+K_i}^{\infty} (V_T - D - K_i) f(V_T) dV_T \ . \tag{A.34}$$

  The above equation states that the present value of the call-option payoff at expiration must correspond to the observed call-option price today, C_0^i , where $i = 1, 2, \ldots, n$ indicates the number of available option contracts. Each option price constraint is weighted by the volume for that option contract over the total volume over all option contracts for the same stock and the same expiry; this weight is denoted here by w_i.

82

- **Normalization Constraint:** The PDF must integrate to 1:

$$1 = \int_{V_T=0}^{\infty} f(V_T)dV_T .$$

(A.35)

In general, the problem is solved by taking the Frechét derivative of the Lagrangian (see the paper for details). The end result is:

$$f(V_T, \lambda) = \frac{1}{\mu(\lambda)}f^0(V_T) \times \exp\Big[\lambda_1 e^{-rT}1_{V_T>D}(V_T - D) +$$

$$\sum_{i=1}^{n} w_i\lambda_{2,i}e^{-rT}1_{V_T>D+K_i}(V_T - D - K_i)\Big]$$

(A.36)

$$\mu(\lambda) = \int_{V_T=0}^{\infty} f^0(V_T) \times \exp\Big[\lambda_1 e^{-rT}1_{V_T>D}(V_T - D) +$$

$$\sum_{i=1}^{n} w_i\lambda_{2,i}e^{-rT}1_{V_T>D+K_i}(V_T - D - K_i)\Big]dV_T$$

(A.37)

where $1_{x>y}$ corresponds to the indicator function that takes the value of 1 whenever $x > y$ and 0 otherwise. To evaluate (A.36), the λs need to be calculated from the constraints. This is equivalent to solving the following system:

$$\frac{1}{\mu(\lambda)}\frac{\partial\mu(\lambda)}{\partial\lambda_1} = E_0$$

(A.38)

$$\frac{1}{\mu(\lambda)}\frac{\partial\mu(\lambda)}{\partial\lambda_{2,i}} = C_0^i \text{ for } i = 1, 2, \ldots, n .$$

(A.39)

This system is highly nonlinear and is solved numerically via Newton minimization. It should be noted that the authors assume a uniform prior distribution.

Overall, when trying to solve for the density, at least 2 option contracts are needed to solve the problem. One contract is used to pin down D while the other is used to shape the density $f^*(V_T, D)$. More contracts are clearly useful to obtain a better representation of the data.

Suppose that two option contracts are available. The contracts are written on the same stock, and expire on the same date. The empirical strategy consists of using one option contract to solve the first optimization problem in (A.32) and using the second option contract to search for the D that is able to price the second option contract. The implementation algorithm is presented below:

1. Calibrate the maximum value that V can take, V_{max}, which is based on the book value of assets, the average growth rate over the last four quarters of the book value of assets, and its standard deviation.

83

2. Starting from a suitable initial guess of D, call it D_0, divide the domain of V into two sub-intervals, $DS^0 = [0; D_0]$ describing the default state, and $NDS^0 = [D_0 + \epsilon; V_{max}]$ indicating the non-default state.

3. When $V_T \in DS^0$, discretize the domain by allowing V_T to take only two values: 0 or D_0 , and start by setting $f^0(V_T = 0) = 0$ and $f^0(V_T = D_0)$, requiring $PoD(D_0) = 0$.

4. When $V_T \in NDS^0$, discretize the domain by constructing 100 equally spaced values that V_T can take. Then assign the same likelihood to each of these values (since the prior is a uniform distribution) so that under $f^0(V_T)$, $Pr(V_T \in NDS^0) = 1 - PoD(D_0) = 1$.

5. Solve numerically for a new D, called D', and $f(V_T, D')$ for which $f(V_T = 0) = 0$, $PoD(D') = f(V_T = D')$, and $Pr(V_T \in NDS') = 1 - PoD(D') = 1 - f(V_T = D')$.

   Note that for the first iteration, this means the following: given that V_T is uniformly distributed in NDS^0, one can solve for D' by using one option contract to numerically solve the equation of the form shown in (A.34); given D', the system of equations (A.36)–(A.39) can be numerically solved for $f(V_T, D')$.

   For subsequent iterations, the new D is found by solving an option contract constraint equation using the density obtained in the previous iteration which is evaluated in the NDS interval determined at the end of the previous iteration. The significance of using the new NDS interval is that there (may) be a mass of probability on the left end. After this new D is obtained, the system of equations (A.36)–(A.39) can be numerically solved for the new density.

6. Once a solution is obtained, repeat steps (3)–(5). However, now fix the range of values V can take between $DS' = [0; D']$ and $NDS' = [D' + \epsilon; V_{max}]$, and determine new values for $f(V_T, D'')$ and D''.

7. The procedure stops when $D'' = D' = D^*$, so that $f^*(V_T, D') = f^*(V_T, D'') = f^*(V_T, D^*)$, and $Pr(V_T \in NDS^*) = 1 - PoD(D^*) = 1 - f^*(V_T = D^*)$.

8. The option-iPoD corresponds to $PoD(D^*)$.

Note that applying the above algorithm can be used to infer a "term structure" of iPoDs since one can use groups of options with different maturities and get an iPoD for each maturity.

C.3.2 Inputs and Outputs

Capuano (2008) estimates the iPoD of 10 U.S. financial institutions: Bank of America, Citigroup, JPMorgan, Wachovia, Wells Fargo, Bear Stearns, Goldman Sachs, Lehman Brothers, Merrill Lynch, and Morgan Stanley. For each bank, the following inputs are needed to generate its iPoD for a given time interval:

- **Input:** All European option prices on a given date t for a given maturity T as well as their volumes and their corresponding strike prices. The author illustrates his method for Citigroup, where the date used was February 12, 2008 and the option expiries were on June 21, 2008. All option data is retrieved from Bloomberg. The volumes are needed to weight the importance of the option pricing constraints, i.e., w_i.

- **Input:** The stock price at the given date t.

- **Input:** The imposed value of V_{max}. The author does not specifically give a formula for determining V_{max}, but from the information given, it seems to be calculated by projecting the time-t book value of assets to time T by using the average growth rate of the past four quarters plus some multiple of its standard deviation.

- **Input:** The prior distribution $f^0(V_T)$, which is assumed to be uniform in NDS^0 and to be zero in DS^0.

- **Output:** The option-iPoD for the bank.

- **Output:** The PDF of the asset value at time T, which can be used to estimate other variables of interest, including the Greeks of the options.

C.3.3 Empirical Findings

First, it should be noted that the framework is not able to describe the PDF in the default state as the inputs are equity option prices, which by definition give no information in the case of default. However, the framework is informative in determining the probability of default and the PDF of the asset value overall, with the only "adjustments" that the PDF is defined for $V \geq D$ and that there is a probability mass at $V = D$ which reflects the probability of default.

In the case of the Bear Stearns collapse, the option-iPoD was able to signal market sentiment early. Using options expiring on March 22, 2008, the closest to the March 14 collapse, the author computes the option-iPoD from February 12 to March 19, 2008 and finds that the option-iPoD started to indicate some market nervousness on February 21. On February 29, the option-iPoD jumped by a factor of 766 with respect to the previous day. The following week a relative calm seemed to return, but on March 10 the iPoD jumped again, a jump 4 times bigger than the previous one. The iPoD reached its peak on March 14, but quickly dropped on March 17, following the rescue plan announced by the Federal Reserve. In addition, during this episode, changes in option-iPoD appeared to be a leading indicator for changes in the level of CDS spreads.

Another paper which uses the minimum cross-entropy approach in deriving probability distributions is Segoviano and Goodhart (2009), which is discussed in Section C.4.

C.4 Multivariate Density Estimators

Segoviano and Goodhart (2009) develop a measure of systemic risk based on the banking system's multivariate density (BSMD) function. Their approach defines the banking system as a portfolio of banks and infers its multivariate density from which the proposed measures are estimated. The proposed BSMDs represent a set of tools to analyze (define) stability from three different, yet complementary perspectives, by allowing the quantification of "common" distress in the banks of a system, distress between specific banks, and distress in the system associated with a specific bank, i.e., "cascade effects". These measures embed the banks' distress interdependence structure, which captures not only linear correlation but also nonlinear distress dependencies among the banks in the system. Moreover, the structure

of linear and nonlinear distress dependencies changes endogenously as banks' probabilities of distress (PoDs) change; hence, the proposed stability measures incorporate changes in distress dependence that are consistent with the economic cycle.

C.4.1 Definition

The banking system's multivariate density characterizes both the individual and joint asset-value movements of the portfolio of banks representing the banking system. It is estimated via the minimum cross-entropy method used also in the option-iPoD method (see Section C.3).

Assume that there are a set of n banks; each bank's log asset returns is x_i. The problem to be solved is:

$$\min_{p(x_1, x_2, \ldots, x_n)} \int \int \cdots \int p(x_1, x_2, \ldots, x_n) \log \left[\frac{p(x_1, x_2, \ldots, x_n)}{q(x_1, x_2, \ldots, x_n)} \right] dx_1 dx_2 \cdots dx_n \qquad \text{(A.40)}$$

where $p(x_1, x_2, \ldots, x_n)$ is the BSMD that we are solving for, $q(x_1, x_2, \ldots, x_n)$ is the prior joint probability density function, and the integrand is the cross-entropy between the prior and the posterior; it represents the degree of uncertainty around the posterior. The minimization problem posed in (A.40) is subject to the following constraints:

- **Probability of Default constraints (PoD):** For each bank, at time t, the BSMD must be consistent with the empirically estimated probabilities of default PoD_t^i. Denoting bank i's default barrier as x_i^d, its PoD can be written as:

$$PoD_t^i = \int \int \cdots \int p(x_1, x_2, \ldots, x_n) 1_{x_i < x_i^d} dx_1 dx_2 \cdots dx_n \qquad \text{(A.41)}$$

  where $1_{x_i < x_i^d}$ is the indicator function which is nonzero when bank i's asset value is below the default barrier x_i^d. Define as λ_i the Lagrange multiplier associated with bank i's PoD constraint.

- **Normalization constraint:** The BSMD must integrate to 1:

$$1 = \int \int \cdots \int p(x_1, x_2, \ldots, x_n) dx_1 dx_2 \cdots dx_n . \qquad \text{(A.42)}$$

Define as μ the Lagrange multiplier associated with this constraint. The resulting Lagrangian

is:

$$L = \int\int\cdots\int p(x_1, x_2, \ldots, x_n)\log\left[\frac{p(x_1, x_2, \ldots, x_n)}{q(x_1, x_2, \ldots, x_n)}\right]dx_1 dx_2 \cdots dx_n +$$

$$\sum_{i=1}^{n}\lambda_i\left[\int\int\cdots\int p(x_1, x_2, \ldots, x_n)1_{x_i < x_i^d}dx_1 dx_2 \cdots dx_n - PoD_t^i\right] +$$

$$\mu\left[\int\int\cdots\int p(x_1, x_2, \ldots, x_n)dx_1 dx_2 \cdots dx_n - 1\right] . \qquad (A.43)$$

The estimated BSMD can be shown to be:

$$p(x_1, x_2, \ldots, x_n) = q(x_1, x_2, \ldots, x_n)\exp\left[-\left(1 + \mu + \sum_{i=1}^{n}\lambda_i 1_{x_i < x_i^d}\right)\right] . \qquad (A.44)$$

In order to evaluate the above expression, the Lagrange multipliers need to be estimated. This can be done by substituting the BSMD expression into the constraints (equations (A.41) and (A.42)) and solving $n + 1$ system of equations for the $n + 1$ unknowns (n λs and μ). Given the BSMD, the authors propose the following measures of systemic risk:

1. **Joint Probability of Default (JPoD):** The joint probability of default, which represents the probability of all the banks in the system becoming distressed, i.e., the tail risk of the system:

$$\text{JPoD} = \int_0^{x_d^1}\int_0^{x_d^2}\cdots\int_0^{x_d^n} p(x_1, x_2, \ldots, x_n)dx_1 dx_2 \cdots dx_n . \qquad (A.45)$$

The JPoD captures changes in the distress dependence among the banks, which increases in times of financial distress; therefore, in such periods, the banking system's JPoD may experience larger and nonlinear increases than those experienced by the (average) PoDs of individual banks.

2. **Banking Stability Index (BSI):** The banking stability index reflects the expected number of banks becoming distressed given that at least one bank has become distressed. A higher number signifies increased instability.

$$\text{BSI} = \frac{\sum_{i=1}^{i=n} Pr(x_i < x_i^d)}{1 - Pr\left(x_1 > x_1^d, x_2 > x_2^d, \ldots, x_n > x_n^d\right)} . \qquad (A.46)$$

3. **Distress Dependence Matrix (DDM):** The distress dependence matrix contains the probability of distress of the bank specified in the row, given that the bank specified

in the column becomes distressed. Thus, the (i, j) element of this matrix is:

$$\Pr\left(x_i < x_i^d \mid x_j < x_j^d\right) = \frac{\Pr\left(x_i < x_i^d, x_j < x_j^d\right)}{\Pr\left(x_j < x_j^d\right)} . \tag{A.47}$$

Although conditional probabilities do not imply causation, this set of pairwise conditional probabilities can provide important insights into interlinkages and the likelihood of contagion between the banks in the system.

4. **Probability of Cascade Effects (PCE):** Given that a specific bank becomes distressed, the probability of cascade effects characterizes the likelihood that one, two, or more institutions, up to the total number of banks in the system, become distressed. Therefore, this measure quantifies the potential "cascade" effects, and the systemic importance of a specific bank, in the system given distress at a specific bank. For example, in a banking system with four banks, X, Y, Z, and R, the PCE, given that X becomes distressed, is:

$$\begin{aligned} \text{PCE} = \ & P(Y|X) + P(Z|X) + P(R|X) - \\ & \left[P(Y \cap R|X) + P(Y \cap Z|X) + P(Z \cap R|X) \right] + P(Y \cap R \cap Z|X) \end{aligned} \tag{A.48}$$

C.4.2 Inputs and Outputs

- **Input:** The PoD of each bank of interest. This can be inferred from various data and the method does not depend on the particular type of PoD. In their paper, the authors use CDS daily data from January 2005 to October 2008 to infer the implied PoDs for each day.

- **Input:** The prior distribution $q(x_1, x_2, \ldots, x_n)$. The authors assume that it is normal with a mean of zero and a covariance matrix equal to the identity matrix. This is consistent with assuming log-normality of the asset values (recall that x_i is the logarithm of bank i's asset returns).

- **Input:** The default barrier for each bank. The authors use the same method as in Segoviano (2006); for bank i, a daily time series of its PoD is constructed. The time series is then averaged to obtain $\bar{\text{PoD}}_i$. The default barrier is then given by:

$$x_d^i = \phi^{-1}\left(-(1 - \bar{\text{PoD}}_i)\right) . \tag{A.49}$$

- **Output:** The multivariate density of the n banks' log asset returns.

- **Output:** The JPoD as defined in (A.45).

- **Output:** The BSI as defined in (A.46).

- **Output:** The DDM as defined in (A.47).

- **Output:** The PCE as defined in (A.48).

C.4.3 Empirical Findings

The authors run their systemic risk analysis on the U.S. banking system, the E.U. banking system, and the "world" banking system. Their key findings are that U.S. banks are highly interconnected as measured by the JPoD and the BSI; furthermore, the movement in these two systemic risk measures coincide with events that were considered relevant by the markets on specific dates. Also, the distress dependence across banks rises during times of crisis, indicating that systemic risks, as implied by the JPoD and the BSI, rise faster than idiosyncratic risks. Links across major U.S. banks increased greatly from 2005 to 2008; on average, if any of the U.S. banks fell into distress, the average probability of the other banks being distressed increased from 27% on July 1, 2007 to 41% on September 12, 2008. On September 12, 2008, Lehman was the bank under the highest stress as measured by its PoD conditional on any other bank falling into distress (56% on average). The European JPoD and BSI move in tandem with movements in the U.S. indicators, also coinciding with relevant market events.

The authors also run their method in assessing the risks to sovereigns arising from banks' distress. One of several interesting results is that distress among Spanish and Italian banks are estimated to be the highest in their respective geographical regions, and that distress at Standard Chartered Bank would imply significant stress in Asia. These results suggest that geography matters.

The BSMD method is applied at the individual bank portfolio level in Basurto and Padilla (2006), and further details on this methodology may be found in Segoviano and Goodhart (2009) and Segoviano (2006).

C.5 Simulating the Housing Sector

Khandani, Lo, and Merton (2009) posit that rising home prices, declining interest rates, and near-frictionless refinancing opportunities led to vastly increased systemic risk in the financial system. A simultaneous occurrence of these factors imposes an unintentional synchronization of homeowner leverage. This synchronization, coupled with the indivisibility of residential real estate that prevents homeowners from deleveraging when property values decline and homeowner equity deteriorates, conspire to create a "ratchet" effect in which homeowner leverage is maintained or increased during good times without the ability to decrease leverage during bad times.

To measure the systemic impact of this ratchet effect, the U.S. housing market is simulated with and without equity extractions, and the losses absorbed by mortgage lenders is estimated by valuing the embedded put option in non-recourse mortgages. The proposed systemic risk indicator for the housing market is the dollar-delta of this embedded put option.

C.5.1 Definition

To value the put-option delta of the total mortgages outstanding, a model needs to be created which is calibrated to be representative of the actual stock of homes in the U.S.

In the simulation, each house enters when it is first sold, and stays until its mortgage is fully paid. In the process, the house may be refinanced one or more times. The decision rule on whether to refinance is a function of the loan-to-value ratio and is calibrated so as to reproduce historical housing data such as the total value of residential mortgages outstanding and the total value of equity extracted from homes. The simulation that the authors run uses the following assumptions:

1. Each house is purchased at an initial LTV_0 of 85%.

2. All homes are purchased with conventional 30-year fixed-rate mortgages that are non-recourse loans.

3. The market value of all houses that are in "circulation", i.e., that have outstanding mortgages and may be refinanced at some point, grows at the rate given by the Home Price Index (HPI).

4. Only national data on home sales, price appreciation, and mortgages outstanding are used to calibrate the simulation.

5. A homeowner's decision to refinance is made each month, and is only a function of the current equity in the home and the potential savings from switching to a lower interest-rate mortgage.

6. For rate refinancing, also called "no-cash-out" refinancing, the owner will refinance when rates have fallen by more than 200 basis points. The new mortgage is assumed to have the same maturity as the existing mortgage, and the principal of the new mortgage is equal to the remaining value of the existing mortgage. Therefore, there is no equity extracted from the house in this case.

7. For cash-out refinancings, the homeowner will refinance to take out the maximum amount of equity possible. Therefore, the LTV ratio will be brought back to LTV_0 after each refinancing, and a new loan with a maturity of 30 years will be originated.

8. The refinancing decisions by homeowners are random and independent of each other, apart from the dependence explicitly parameterized in the refinancing rule.

9. Once fully owned, a home will not re-enter the market.

The total value of all homes at time t is the total value of all homes that were either directly purchased at time t or were purchased in an earlier period, survived until time $t-1$, and cash-out refinanced at time t.

$$\text{TOTALV}_t \;\; = \;\; \text{NH}_t \times \text{VALUE}_{t,t} \; + \; \sum_{i=1}^{t-1} \text{TOTALV}_i \times \text{SURVIV}_{i,t-1} \times \text{REFI}_{i,t} \times \frac{\text{VALUE}_{i,t}}{\text{VALUE}_{i,i}}$$
$$(\text{A.50})$$

where NH_t is the number of new homes entering the system in vintage t, $\text{SURVIV}_{i,t-1}$ is the probability that a new home from vintage i has not undergone a cash-out refinancing by time $t-1$, and $\text{REFI}_{i,t}$ is the probability that a home entering the system in vintage i

90

undergoes a cash-out refinancing at date t, conditioned on the event that it has not yet been refinanced by date t. The variable $\text{VALUE}_{i,t}$ is the value of a house from vintage i by time t, where $i \leq t$. Thus, $\text{VALUE}_{i,i} = \text{NHP}_i$, where NHP_i is the average price for all new homes sold in vintage i. The value for all subsequent periods grows at the rate given by the HPI. Thus, the multiplier $\text{VALUE}_{i,t}/\text{VALUE}_{i,i}$ is an adjustment factor that reflects time variation in housing prices.

The authors find that the following decision rule calibrates the simulation well to the history of outstanding mortgage volume and to historical equity extractions:

$$\text{REFI}_{i,t} \;=\; \begin{cases} 0\% & \text{if } \text{LTV}_{i,t} = 0 \\ 0.3\% & \text{if } t \leq 1988 \text{ and } \text{LTV}_{i,t} \in (0\%, 85\%] \\ 0.9\% & \text{if } t \geq 1989 \text{ and } \text{LTV}_{i,t} \in (0\%, 85\%] \end{cases} . \tag{A.51}$$

Thus, before 1989, the probability of refinancing is 0.3% for homes with LTV less than 85%. After 1989, there is a structural break and the probability of refinancing is higher at 0.9% for homes with LTV less than 85%. Given this decision rule, the variables $\text{SURVIV}_{i,t-1}$ and $\text{REFI}_{i,t}$ are determined and thus TOTALV_t in (A.50) can be evaluated. The above simulation is called the "cash-out" refinancing simulation.

To understand the effects of the cash-out refinancing simulation above, the authors also run a simulation where only rate refinancing occurs as described in Assumption 6 above; this is called the "no-cash-out" refinancing simulation. It is equivalent to evaluating (A.50) where $\text{REF}_{i,t} = 0$ and $\text{SURVIV}_{i,t} = 1$ for all i and t. Clearly, this is not realistic but it is the baseline scenario of risk due to the fact that the dynamics of LTVs are driven purely by home price changes as opposed to the cash-out refinancing case, where equity extractions play an important role in the dynamics of LTVs.

To evaluate the value of the embedded put option on the total value of homes outstanding, a stochastic process for housing prices needs to be specified. The authors assume that housing prices follow a discrete-time geometric random walk represented by a recombining tree and that markets are dynamically complete so that options on property values can be priced by no-arbitrage arguments alone. The put option is assumed to be "Bermudan", meaning that it can be exercised on a set of fixed dates: these exercise dates are once a month, just prior to each mortgage payment date. The exercise price is the amount of principal outstanding on the loan, which declines each month due to the monthly mortgage payments. Furthermore, a volatility of 8% and a rental "yield" of 4% are assumed. For a detailed explanation of binomial tree option pricing methods, see Cox and Rubinstein (1985); a more general discussion of option pricing can be found in Hull (2000).

Denote by $\text{GRT}_{i,t}$ the value of the guarantee, i.e., the put option, calculated for a single home from vintage i at time t. Then, the aggregate value of the guarantees is:

$$\text{TOTALGRT}_t \;=\; \sum_{i=1}^{t} \text{TOTALV}_i \times \text{SURVIV}_{i,t} \times \frac{\text{GRT}_{i,t}}{\text{VALUE}_{i,i}} . \tag{A.52}$$

If TOTALGRT$_t$ is used to measure the potential losses realized after a housing market downturn, then its sensitivity to changes in home prices can serve as a measure of systemic risk. This sensitivity can be easily derived from the embedded option's "delta", i.e., partial derivative with respect to the price of the underlying asset. Let DLT$_{i,t}$ be the sensitivity of the value of the guarantee, GRT$_{i,t}$, to changes in the price of real estate for a single home from vintage i at time t. Then, the aggregate sensitivity, denoted by TOTALDLT$_t$, is given by:

$$\text{TOTALDLT}_t \;\; = \;\; \sum_{i=1}^{t} \text{TOTALV}_i \times \text{SURVIV}_{i,t} \times \frac{\text{DLT}_{i,t}}{\text{VALUE}_{i,i}} \; . \tag{A.53}$$

Thus, TOTALDLT$_t$ measures the dollar change in the aggregate value of guarantees of non-recourse mortgages given an incremental change in the value of the underlying real estate.

The ratio of the TOTALDLT$_t$ for the cash-out refinancing simulation compared to the no-cash-out simulation is proposed as a measure of systemic risk.

C.5.2 Inputs and Outputs

The authors use housing market monthly data from January 1919 to December 2008.

- **Input—Home Price Index (HPI):** The authors use three sources to assemble this series. For the most recent history (since January 1987), they use the S&P/Case-Shiller Home Price Composite (available at Standard and Poor's website). From 1975:Q1 to 1986:Q4, they use the national house price index from the FHFA (available in the "All Transactions Indexes" section of http://www.fhfa.gov/Default.aspx?Page=8). Prior to 1975:Q1, they use the nominal home price index collected by Robert Shiller (available at http://www.econ.yale.edu/~shiller/data.htm). These last two series are only available at a quarterly and annual frequency, respectively, and to be consistent with the rest of the simulation, they are converted into monthly series assuming geometric growth.

- **Input—New Homes Entering the Mortgage System (NH):** This time series is constructed from a variety of sources. The time series of "New One-Family Houses Sold" available from the U.S. Census Bureau is the starting point.[47] This series is available monthly since January 1963. However, it only includes homes built for sale and, for example, excludes homes built by homeowners and contractors. To take such cases into account, the authors use data collected by the U.S. Census Bureau on the intent of completed home constructions.[48] This construction data separates the completed units by their intent: units in the "Built for Sale" category correspond to homes that will be reported in the "New One-Family Houses Sold" upon the completion of a sale transaction. The authors take the sum of construction numbers reported under the "Contractor-Built", "Owner-Built", and "Multi-Units Built for Sale" categories, and use the ratio of this sum to the number of "One-Family Units Built for Sale" to

---

[47]http://www.census.gov/const/www/newressalesindex.html

[48]See the annual series of "Quarterly Housing Completions by Purpose of Construction and Design Type" at http://www.census.gov/const/www/newresconstindex_excel.html, which goes back to 1974.

adjust the "New One-Family Houses Sold" series. For the period from 1963 to 1973, this ratio is not available, so they use the average of the adjustment factor from 1974 to 1983 to make the adjustments prior to 1974. This yields values of NH_t back to January 1963. For dates before 1963, the authors run a statistical analysis relating housing supply to real home prices and population; Appendix A2 of their paper gives details on this.

- **Input—New House Prices (NHP):** The authors use the average home price available from the U.S. Census Bureau for "New One-Family Houses Sold" in the period from January 1975 to December 2008. From January 1963 to December 1974, the authors multiply the reported median prices in the sample by 1.05. From January 1919 to December 1962, they use the growth rate of HPI_t to extrapolate sales prices, starting with the sales price in January 1963.

- **Input—Long-Term Risk-Free Rates (RF):** From February 1977 to December 2008, the 30-year constant maturity UST. The gap between March 2002 and January 2006 is filled using the 20-year constant maturity UST. For the period prior to February 1977, the annual "Long Rate" collected by Robert Shiller is used.[49] The authors use linear interpolation to obtain monthly observations.

- **Input—Mortgage Rates (MR):** The authors use the series constructed by Freddie Mac for the 30-year fixed-rate mortgage rate, which starts in April 1971.[50] For the earlier period, the authors add 150 basis points to the long-term risk-free rates (RF).

- **Input:** The authors use the Matlab (version 7.2) Financial Derivatives Toolbox (Version 4.0) and the functions: crrtimespec, crrsens, crrtree, instoptstock, intenvset, and stockspec.

- **Output:** The total value of mortgage-lender guarantees (see (A.52)).

- **Output:** The ratio of the $TOTALDLT_t$ for the cash-out refinancing simulation compared to the no-cash-out simulation, where $TOTALDLT_t$ is given by (A.53).

C.5.3 Empirical Findings

The process of equity extraction causes the width of the buffer between house value and option strike price to decrease, resulting in a larger portion of the losses transferred to the equity-holders and debt-holders of various lending entities. The authors' simulations show that with the downturn in the value of residential real estate in 2007 and 2008, the value of the guarantees extended to homeowners by mortgage lenders increased substantially up to $1,543 billion, which is too large to be absorbed by the equity of these lending entities alone, creating the need for government intervention to address these losses.

As for the ratio of $TOTALDLT_t$ between the cash-out refinancing case and the no-cash-out case, it increased steadily over the course of the housing boom from the late 1980s until the peak in mid-2006. According to this measure, just prior to that peak in June 2006,

---

[49]See http://www.econ.yale.edu/~shiller/data.htm.
[50]See http://www.freddiemac.com/dlink/html/PMMS/display/PMMSOutputYr.jsp.

cash-out equity extractions increased the magnitude of losses in the event of a decline in real-estate prices by a factor of 5.5. Moreover, while the magnitude of this delta-based measure of systemic risk exposure to the housing market reached its highest level in early 2006, its lowest level since 1990 was only slightly below 4. In other words, cash-out equity extractions will greatly multiply the potential losses from any real-estate market downturn, so that even a small decline in housing prices is magnified into much larger losses for mortgage lenders. And after an extended period of rising home prices, falling interest rates, and refinancings, the ratchet effect can create so much irreversible leverage in the housing market that the question is no longer *if* large-scale defaults will occur, but rather *when* they will occur. The embedded options in non-recourse mortgages also imply that convexity plays a role in that the losses on mortgages experienced by lenders from an initial housing price decline will be substantially smaller than subsequent losses if prices were to decline again by the same dollar amount.

C.6   Consumer Credit

Khandani, Kim, and Lo (2010) apply machine-learning techniques to construct nonlinear nonparametric forecasting models of consumer credit risk. By combining customer transactions and credit bureau data from January 2005 to April 2009 for a sample of a major commercial bank's customers, they are able to construct out-of-sample forecasts that significantly improve the classification rates of credit-card-holder delinquencies and defaults. The authors use a "machine-learning" approach in creating their consumer credit risk forecast model. They subsequently aggregate forecasts for individual accounts by tabulating the number of accounts forecasted to be delinquent over a given forecast period. The proportion of forecasted delinquencies in the population is then used as a systemic risk indicator for consumer lending.

C.6.1   Definition

The objective of any machine-learning model is to identify statistically reliable relationships between certain features of the input data and the target variable or outcome. The features the authors use include data items such as total inflow into a consumer's bank account, total monthly income, credit-card balance, etc., and the target variable is a binary outcome that indicates whether an account is delinquent by 90 days or more within the subsequent 6- or 12-month window. The complete list of input variables the authors use can be found in Table A.4.

The authors use generalized classification and regression trees (CART) (see Breiman, Friedman, Olshen, and Stone (1984)) to construct their forecast models. CART is a widely used statistical technique in which a dependent or "output" variable (either continuous or discrete) is related to a set of independent or "input" variables through a recursive sequence of simple binary relations. CART models can easily be applied to problems with high dimensional feature spaces. Suppose that there are N observations of the dependent variable $\{y_1, \ldots, y_N\}$: in this case each y_i is a binary variable which is 1 if customer i defaults over the next period (6 or 12 months) and zero otherwise. Furthermore, the corresponding D-dimensional feature vector is denoted as $\{\mathbf{x_1}, \ldots, \mathbf{x_N}\}$, where D is the number of model input characteristics listed in Table A.4. The parameters of the CART model are estimated on

Model Inputs

Credit Bureau Data

Total number of trade lines
Number of open trade lines
Number of closed trade lines
Number and balance of auto loans
Number and balance of credit cards
Number and balance of home lines of credit
Number and balance of home loans
Number and balance of all other loans
Number and balance of all other lines of credit
Number and balance of all mortgages
Balance of all auto loans to total debt
Balance of all credit cards to total debt
Balance of all home lines of credit to total debt
Balance of all home loans to total debt
Balance of all other loans to total debt
Balance of all other lines of credit to total debt
Ratio of total mortgage balance to total debt
Total credit-card balance to limits
Total home line of credit balances to
limits
Total balance on all other lines of credit to limits

Transaction Data

Number of Transactions
Total inflow
Total outflow
Total pay inflow

Total food-related expenses
Total grocery expenses
Total restaurant expenses
Total fast-food expenses
Total bar expenses

Transaction Data (cont.)

Total expenses at discount stores
Total expenses at big-box stores
Total recreation expenses
Total clothing store expenses
Total department store expenses
Total other retail store expenses

Total utilities expenses
Total cable TV & Internet expenses
Total telephone expenses
Total net flow from brokerage account
Total net flow from dividends and annuities

Total gas station expenses
Total vehicle-related expenses

Total lodging expenses
Total travel expenses
Total credit-card payments

Total mortgage payments

Total outflow to car and
student loan payments
Total education-related expenses

Deposit Data

Checking account balance
CD account balance
Brokerage account balance
Savings account balance

Table A.4: The inputs used in the machine-learning model of consumer credit risk

the training dataset by recursively selecting features from $\mathbf{x} \in \{x_1, \ldots, x_D\}$ and parameters $\{L_j\}$ that minimize the residual sum-of-squared errors. The "pruning criterion" for stopping the expansion of the tree so as to avoid overfitting the training data is the Gini measure:

$$G(\tau) \;\; = \;\; \sum_{k=1}^{K} P_\tau(k)(1 - P_\tau(k)) \tag{A.54}$$

where τ refers to a leaf node of a CART model and $P_\tau(k)$ refers to the proportion of training data assigned to class k at leaf node τ. Then the pruning criterion for CART model T is defined as:

$$C(T) \;\; \equiv \;\; \sum_{\tau=1}^{|T|} G(\tau) \;\; + \;\; \lambda|T| \tag{A.55}$$

where $|T|$ refers to a number of leaf nodes in CART model T, and λ refers to a regularization parameter chosen by cross-validation. Once the pruning criterion reaches the minimum, the CART algorithm will stop expanding the tree.

In order to improve the forecast power of the CART model, the authors employ an "adaptive boosting" technique as in Freund and Shapire (1996). Instead of equally weighting all the observations in the training set, they weigh the scarcer observations more heavily than the more populous ones. The weights are adjusted recursively as a function of the goodness-of-fit. In particular, the weight for observation i at the nth iteration of CART is given by:

$$w_i^n \;\; = \;\; w_i^{n-1} \exp[\alpha_{n-1} I(f_{n-1}(\mathbf{x}_i) \neq y_i)] \tag{A.56}$$

and the data re-weighting coefficient α_{n-1} is defined as:

$$\alpha_{n-1} \;\; \equiv \;\; \ln\left(\frac{1 - \epsilon_{n-1}}{\epsilon_{n-1}}\right) \tag{A.57}$$

where $I(\cdot)$ is an indicator function that indicates whether the model has correctly predicted the outcome y_i given the input vector x_i, and ϵ_{n-1} is the weighted average error of the model from the $(n-1)^{\text{th}}$ iteration.

To understand how the timing of the training of the model works, Table A.5 depicts the logic for a particular 6-month prediction case. For each evaluation period, the model is calibrated on credit-card delinquency data over the 6-month period specified in the Training Window, and predictions are based on the data available as of the date in the Prediction Date row. More specifically, the model is calibrated using input data, i.e., the feature vector, available in January 2008 and credit-card delinquency data from February to July 2008; note that all transaction data inputs are averaged over the previous 6 months, i.e., August 2007 to January 2008. The resulting "trained" model is applied to the July 2008 feature vector

to generate forecasts of delinquencies from August 2008 to January 2009. Thus, for each customer i, the output for the example in Table A.5 is a "probability" that he will go 90+ days delinquent between August 2008 and January 2009. The above sequence is run every month, meaning that the next training window would start in March 2008.

Dates	Description of Phase
Feb-08	Training Window Start
Jul-08	Training Window End
Jul-08	Prediction Date
Aug-08	Evaluation Window Start
Jan-09	Evaluation Window End

Table A.5: Description of the phases in estimating a machine-learning model for the 6-month prediction case.

A customer's delinquency probability can be classified into a two-state prediction ("delinquent" or "current") by comparing his forecasted probability of delinquency to a threshold: if his forecasted probability is higher than the threshold, the prediction is that he will be "delinquent". If not, the prediction is that he will be "current". Clearly, there is a trade-off involved when setting the threshold: decreasing the threshold risks increasing the number of false positives and increasing the threshold risks decreasing the number of true positives. The authors set the threshold such that this trade-off is one-to-one. Mechanically, this means searching over thresholds between 0% and 100% such that a small increase in the threshold leads to equal and opposite percent movements in the true positive and false positive rates. Given the above refinement of classifying each customer as one who will default or not in the future period of interest, one can estimate the predicted 90-plus-days delinquency rate over the future period of interest.

C.6.2   Inputs and Outputs

- **Input:** The monthly data listed in Table A.4 between January 2005 to April 2009. This data is proprietary in nature and temporary access was given to MIT researchers by a major commercial bank.

- **Output:** The predicted 90-plus-day delinquency rate on credit cards over the future 6- and 12-month period.

C.6.3   Empirical Findings

The authors find that forecasted delinquencies are highly correlated with realized delinquencies with linear regression R^2 of 85% for both the 6- and 12-month horizons; hence the forecasts are capturing the dynamics of the consumer credit cycle over the sample. However, they find that the forecasts consistently underestimate the absolute level of delinquencies by a scalar multiple, which they attribute to an artifact of the particular classification threshold selected. To correct for this underestimation, and to reduce the effects of overfitting, the authors use 10-fold cross-validation to estimate the model's recall value (the ratio of true

negatives to the sum of true negatives and false positives). For a given training window, this consists of stratifying the dataset into 10 bins, using 9 of them for training of the model and the remaining one for testing, and repeating this step 9 times using a different set of bins for testing and training. The model's recall value is estimated by averaging the 10 resulting recall values. After obtaining the model's recall value for a given horizon, the authors multiply the forecasted default rate for that horizon by the reciprocal of this estimated recall rate. This is reasonably successful in correcting the underestimation bias, yielding forecasts that are accurate both in terms of level and dynamics.

C.7 Principal Components Analysis

Kritzman, Li, Page, and Rigobon (2010) propose to measure systemic risk via the Absorption Ratio (AR), which they define as the fraction of the total variance of a set of asset returns explained or "absorbed" by a fixed number of eigenvectors. The absorption ratio captures the extent to which markets are unified or tightly coupled. When markets are tightly coupled, they become more fragile in the sense that negative shocks propagate more quickly and broadly than when markets are loosely linked. The authors apply their AR analysis to several broad markets, introduce a standardized measure of shifts in the AR, and analyze how these shifts relate to changes in asset prices and financial turbulence.

C.7.1 Definition

In order to calculate the AR at a given time, the $N \times N$ covariance matrix of the N asset returns is needed. Mathematically, the AR is defined as:

$$\text{AR} \;=\; \frac{\sum_{i=1}^{n} \sigma_{E_i}^2}{\sum_{j=1}^{n} \sigma_{a_j}^2} \tag{A.58}$$

where:

$$
\begin{aligned}
n &= \text{number of eigenvectors used in calculating AR} \\
\sigma_{E_i}^2 &= \text{variance of eigenvector } i \\
\sigma_{a_j}^2 &= \text{variance of asset } j
\end{aligned}
$$

A high value for the absorption ratio corresponds to a high level of systemic risk because it implies the sources of risk are more unified. A low absorption ratio indicates less systemic risk because it implies the sources of risk are more disparate. High systemic risk does not necessarily lead to asset depreciation or financial turbulence. It is simply an indication of market fragility in the sense that a shock is more likely to propagate quickly and broadly when sources of risk are tightly coupled.

In order to estimate the AR, the authors use a 500-day window to estimate the covariance matrix and eigenvectors, and fix the number of eigenvectors at approximately $1/5^{\text{th}}$ the number of assets. It should be noted that an eigenvector may or may not be associated with an observable financial variable; it may reflect a combination of several influences that

came together in a particular way, in which case the factor may not be definable as anything other than a statistical artifact. Moreover, the composition of eigenvectors may not persist through time as the sources of risk may change from period to period. But the goal of the AR is not to interpret sources of risk; rather it seeks to measure the extent to which sources of risk are becoming more or less compact.

The authors also propose a technical indicator of AR movements which they find to be a leading indicator of trouble for asset prices:

$$\Delta AR \quad = \quad \frac{\bar{AR}_{\text{15-Day}} - \bar{AR}_{\text{1-Year}}}{\sigma_{AR_{\text{1-Year}}}} \tag{A.59}$$

where:

$$
\begin{aligned}
\Delta AR \quad &= \quad \text{standardized AR shift} \\
\bar{AR}_{\text{15-Day}} \quad &= \quad \text{15-day moving average of AR} \\
\bar{AR}_{\text{1-Year}} \quad &= \quad \text{1-year moving average of AR} \\
\sigma_{AR_{\text{1-Year}}} \quad &= \quad \text{standard deviation of the 1-year AR}
\end{aligned}
$$

C.7.2   Inputs and Outputs

- **Input:** Daily returns for the 51 industries of the MSCI U.S. index from January 1, 1998 to January 31, 2010. The covariance matrices are estimated using a 500-day rolling window of data. This method is very general and can be tailored according to the application. The authors also apply it to other countries, including Canada, Germany, Japan, and the U.K. For each country, they use the daily returns of all industries of the corresponding MSCI index for the period January 1, 1998 to January 31, 2010. The authors also apply the method to the 14 subindexes of the Case-Shiller U.S. housing price index from January 1987 to December 2009.

- **Output:** AR and ΔAR for each case.

C.7.3   Empirical Findings

The authors find evidence that their measures of systemic risk effectively capture market fragility. When using the returns of the 51 U.S. industries in the MSCI USA index as inputs, they find that the AR is highly negatively correlated with stock prices and that all of the 1%-most-significant U.S. stock market drawdowns followed spikes in the AR as measured by ΔAR within a month. The performance is also quite good in predicting the worst 2% and 5% stock market drawdowns. Moreover, on average, stock prices appreciated significantly in the wake of sharp declines in the AR.

When using the 14 subindexes of the Case-Shiller index for housing prices in determining the AR, they find that it was a leading indicator of the U.S. housing market bubble.

They also find that the AR systematically rose in advance of market turbulence, where turbulence is defined as in Kritzman and Li (2010) and whose summary can be found in Section C.2.

Another study which uses PCA as its primer is Billio, Getmansky, Lo, and Pelizzon (2010), which is presented in Section B.5. A good reference for more reading on the mathematics behind PCA is Jolliffe (2002).

D Stress Tests

Stress tests are a special case of forward-looking analysis with a significant role in systemic risk monitoring. Stress testing is codified in regulation and international standards, including the Basel accord. Because the salient issues in stress testing are summarized in the main text (see Section 2.5), we do not repeat that discussion here. Instead, we move directly to a description of specific models.

D.1 GDP Stress Tests

Alfaro and Drehmann (2009) propose a macroeconomic stress test using a simple AR model of GDP growth which is assumed to depend only on its past behavior. The reason the authors focus on GDP is they observe that domestic macroeconomic conditions as measured by GDP growth typically weaken ahead of banking crises. Furthermore, output drops substantially in nearly all of the observed crises once stress emerges. Their approach to stress testing uses this as a starting point to construct GDP scenarios, independently of whether falls in output truly reflect or cause crises. Their guiding principle in creating their GDP shocks is that stress scenarios should be severe yet plausible.

D.1.1 Definition

In defining crises, the authors rely on Reinhart and Rogoff (2008), who define the beginning of a crisis by the emergence of large-scale policy assistance or the default of important players in the financial system.

The authors assume an AR model of GDP growth (to replicate the information available to policymakers before a crisis, the authors estimate a different model for each crisis in each country, using only data up to the crisis itself):

$$y_t \ = \ \rho_1 y_{t-1} \ + \ \rho_2 y_{t-2} \ + \ \epsilon_t \tag{A.60}$$

where y_t denotes the real GDP growth rate at time t. For each country, the Bayesian Information Criterion (BIC) is used in determining whether 1 or 2 lags should be used. As a stress scenario, the authors use the worst negative forecast error of the above AR model, regardless of whether this coincided with a banking crisis or not. They then shock the model in (A.60) with this very negative ϵ four quarters before the beginning of the crisis and compare the maximum drop in GDP growth during the stress test with the maximum drop during the actual episode. The maximum drop in GDP growth during a crisis is calculated as the difference between the GDP growth four quarters prior to the crisis and the minimum GDP growth within two years after the crisis.

D.1.2 Inputs and Outputs

- **Input:** The set of crises the authors use are:

Argentina	1989:Q4,1995:Q1, 2001:Q4	Malaysia	1997:Q3
Australia	1989:Q4	Mexico	1994:Q4
Belgium	2008:Q3	the Netherlands	2008:Q3
Brazil	1990:Q1	New Zealand	1987:Q1
Canada	1983:Q4	Norway	1991:Q4
Denmark	1987:Q4	the Philippines	1983:Q4, 1997:Q3
Finland	1991:Q3	Singapore	1982:Q4
France	1994:Q1, 2008:Q3	South Africa	1977:Q4
Germany	2007:Q3	Spain	1977:Q4, 1993:Q4, 2008:Q3
Iceland	1985:Q4, 1993:Q4, 2008:Q3	Sweden	1991:Q3
Indonesia	1997:Q4	Switzerland	2007:Q4
Ireland	2008:Q3	Thailand	1997:Q3
Italy	1990:Q3	Turkey	2000:Q4
Japan	1997:Q4, 2008:Q3	United Kingdom	1974:Q4, 1991:Q4, 2007:Q2
Korea	1997:Q3	United States	1988:Q4, 2007:Q3

  A crisis is only included when there is sufficient real GDP growth data preceding it (at least 16 quarters of data).

- **Input:** Quarterly real GDP growth data since 1970:Q1 for all countries with crises listed above.

- **Output:** The stress scenario to be used for each country, which is the worst negative forecast error of that country's AR model.

D.1.3 Empirical Findings

The authors find that in nearly 70% of all cases, the hypothetical stress scenarios fall short of the severity of actual events. Interestingly, for none of the 11 countries that have experienced a banking crisis after 2007 do the stress tests anticipate the severe drop in GDP growth, even though several of these economies had previously experienced crises.

However, stress tests seem to be a useful tool to gauge the potential impact of further adverse shocks if macro conditions are already weak. In 64% of all low-growth episodes,[51] stress scenarios are severe enough. This contrasts starkly with high-growth episodes, where in over 80% of all crises a stress test could not have generated the actual sharp decline in GDP growth.

For an overview of macro stress testing models of various complexity, see Drehmann (2009).

---

[51]Low-growth crises are defined as those preceded by periods of low growth, i.e., annual GDP growth of less than 2% on average in the three years prior to the crisis. High-growth episodes are then defined as crises that are not classified as low-growth crises.

D.2 Lessons from the SCAP

Hirtle, Schuermann, and Stiroh (2009) describe some of the lessons learned from the Supervisory Capital Assessment Program (SCAP) or "stress tests" conducted by the Federal Reserve in 2009–2010 on the 19 largest U.S. bank holding companies. The SCAP is one example of how the macro- and microprudential perspectives can be joined to create a stronger supervisory framework that addresses a wider range of supervisory objectives. The authors review the key features of the SCAP and discuss how they can be leveraged to improve bank supervision in the future.

The goal of macroprudential regulation is to reduce the probability of distress for the entire financial system when that distress has the potential to adversely impact the real economy. This link incorporates a host of potential channels including interdependence and linkages among large financial firms through clearing and settlement systems, common exposures, collective or "herd" behavior, and market failures such as externalities or moral hazard, all of which have the potential to amplify shocks and spill over to the real economy. Supervisors then have an incentive to "lean against the wind" of broader destabilizing forces with countercyclical pressures. The SCAP had important features of macroprudential and microprudential supervision. From the macroprudential perspective, the SCAP was a top-down analysis of the largest bank holding companies (BHCs), representing a majority of the U.S. banking system, with an explicit goal to facilitate aggregate lending. The SCAP applied a common, probabilistic scenario analysis for all participating BHCs and looked beyond the traditional accounting-based measures to determine the needed capital buffer. The macroprudential goal was to credibly reduce the probability of the tail outcome, but the analysis began at the microprudential level with detailed and idiosyncratic data on the risks and exposures of each participating BHC. This firm-specific, granular data allowed tailored analysis that led to differentiation and BHC-specific policy actions.

Supervisors collected significant amounts of confidential data about BHCs' loan and securities portfolios, trading accounts, derivatives positions, and revenue and expense sources. Using these data, supervisors were able to develop independent estimates of losses and revenues for the participating firms. The final SCAP projections of losses, revenues, and reserve needs were developed through review and analysis by more than 150 senior supervisors, examiners, economists, financial analysts, and experts in law, accounting, and regulatory capital at the supervisory agencies.

Some other key elements of the SCAP that made it successful include comprehensive coverage of participating BHCs; use of a consistent framework across firms; use of multiple, independent estimates of loss and revenue, rather than a single model or approach; transparency about the process and results; and a clear set of goals and resulting policy actions that were articulated in advance and well-understood by all parties.

D.3 A 10-by-10-by-10 Approach

In Duffie's (2011) proposal, a regulator would collect and analyze information concerning the exposures of N "important" institutions to M defined stress scenarios. For each stress scenario, an important institution would report its gain or loss, in total, and with respect to its contractual positions with each of the entities for which its exposure, for that scenario, is among the K greatest in magnitude relative to all counterparties. Those counterparties

would be identified, stress by stress. The N significant entities would be those identified by the Financial Stability Board.

With this approach, the joint exposure of the system to particular stress tests and particular entities (or chains of entities) could be clarified as a result. New systemically important entities might emerge from the analysis because some of the K entities to whom a reporting important institution has its largest exposures, for a given stress scenario, need not be among the original N reporting important institutions, but could become identified as systemically vulnerable as a result of the analysis.

The types of stresses that would be explored should involve scenarios that are not covered by delta-based hedging and are conjectured to have potential systemic importance. The sizes of the defined stresses should therefore capture movements that are extreme but plausible. The asset classes covered by the scenarios should be broad enough to incorporate likely increases in cross-asset return correlations in crisis settings.

Illustrative examples of such stress scenarios that the author gives are:

1. The default of a single entity.
2. A 4% simultaneous change in all credit yield spreads.
3. A 4% shift of the U.S.-dollar yield curve.
4. A 25% change in the value of the dollar relative to a basket of major currencies.
5. A 25% change in the value of the Euro relative to a basket of major currencies.
6. A 25% change in a major real estate index.
7. A 50% simultaneous change in the prices of all energy-related commodities.
8. A 50% change in a global equities index.

The results of such a survey can help increase the transparency of hedge funds and how they play into the systemic risk in the financial sector. Theoretically, if a hedge fund failure affects many banks, that hedge fund poses a large systemic risk. Furthermore, if a hedge fund is consistently one of the top gain/loss counterparties with important institutions, it must be systemically important.

A major shortcoming of this approach is that it will miss widely dispersed potential sources of systemic risk that do not flow through major financial institutions. For example, the U.S. Savings and Loans Crisis probably did not register directly measurable stress to just a handful of important financial institutions.

D.3.1 Inputs and Outputs

The author did not actually run any simulations, he just outlined the basic idea. We summarize the inputs and outputs that would be needed to implement his proposal.

- **Input:** The M stress scenarios to be considered by the N important institutions. The author gives examples of such scenarios (see the previous section).

- **Input:** The responses of each of the N important institutions to each of the M stress scenarios. This is survey data and could be made a mandatory reporting requirement for the important institutions on a quarterly basis.

- **Output:** For each important institution and for each stress scenario, a list of the top K counterparties that would suffer.

- **Output:** We propose an extension to the author's measure, which is a "vulnerability index" for institutions that are not part of the N important institutions (for lack of a better term, we call them "non-important institutions"), and suffer or gain a great deal over different stress scenarios in their contractual obligations with the N important institutions. Assume institution k is not part of the N important institutions: for a given scenario m, we define its vulnerability index as:

$$V_{k,m} = \sum_{i=1}^{N} l_{k,i}^{m} \qquad (A.61)$$

where $l_{k,i}^{m}$ denotes the non-important institution's loss, i.e., important institution i's gain, for stress scenario m if non-important institution k is among the important institution i's top K exposures for scenario m. Note that if $l_{k,i}^{m}$ is negative, this reflects a gain for the non-important institution and a loss for the important institution. $l_{k,i}^{m}$ is zero if the non-important institution is not among important institution i's top K exposures for scenario m.

The more positive the vulnerability index is for non-important institution k under stress m, the more it is exposed to stress scenario m. The more negative the vulnerability index is for non-important institution k under stress m, the more of a systemic risk it *poses* to the N important financial institutions under stress scenario m.

E   Cross-Sectional Measures

A complementary philosophy to the forward-looking measure described in Section C is the cross-sectional measure: such an approach aims to examine the co-dependence of institutions on each other's "health". Examples in this class include Adrian and Brunnermeier (2010)'s conditional value at risk (CoVaR) model, and the closely related Co-Risk measure proposed by the International Monetary Fund (2009a). In its simplest form, CoVaR relates two institutions, and is defined as the VaR of one institution at a specific probability quantile (e.g., 99%), *conditional* on the other institution being at its VaR threshold for the same quantile. In other words, a high CoVaR means the first institution tends to accompany the other one into trouble. The relationship need not be symmetric; for example, distress at a SIFI might disrupt a small correspondent bank while troubles at the latter do not materially affect the former. For systemic risk measurement, one simply treats the financial system as one large institution when applying the CoVaR technique. The Co-Risk measure of International Monetary Fund (2009a) is similar in structure to CoVaR, except that Co-Risk examines the CDS spread of one firm, conditional on the CDS spread of the other, each at the respective 95th percentile of its empirical distribution. Adrian and Brunnermeier (2010) use quantile regressions to capture the empirical relationship between VaRs in the tails of the joint distribution (as an alternative to least-squares regressions, which focus on the relationship at the mean of the distribution). External factors enter the model to accommodate the possibility that the joint distribution is evolving over time. Adrian and Brunnermeier estimate the conditional distribution as a function of state variables, including interest rates, credit spreads, equity

prices, and a volatility index. The International Monetary Fund (2009b) adopts a similar procedure for their Co-Risk model, without credit spreads.[52]

E.1 CoVaR

Adrian and Brunnermeier (2010) propose to measure systemic risk via the conditional value-at-risk (CoVaR) of the financial system, conditional on institutions being in a state of distress. An institution's contribution to systemic risk is defined as the difference between CoVaR conditional on the institution being in distress and CoVaR in the median state of the institution. The CoVaR systemic risk measure is able to identify the risk on the system by individually "systemically important" institutions, which are so interconnected and large that they can cause negative risk spillover effects on others, as well as by smaller institutions that are "systemic" when acting as part of a herd. Furthermore, CoVaR is a measure which does not rely on contemporaneous price movements and thus can be used to anticipate systemic risk. The CoVaR measure captures institutional externalities such as "too big to fail", "too interconnected to fail", and crowded trade positions.

E.1.1 Definition

Recall that the value-at-risk of institution i at the q percentile is defined as:

$$\Pr(X^i \leq \text{VaR}_q^i) \;\; = \;\; q \tag{A.62}$$

where X^i denotes the asset return value of institution i. Note that VaR_q^i is typically a negative number. The VaR of institution j (or the financial system) conditional on the event $\{X^i = VaR_q^i\}$, i.e., institution i's asset-return attains its VaR value, is denoted by $\text{CoVaR}_q^{j|i}$, where q is the quantile. Mathematically:

$$\Pr(X^j \leq \text{CoVaR}_q^{j|i} | X^i = \text{VaR}_q^i) \;\; = \;\; q \; . \tag{A.63}$$

Institution i's contribution to the risk of j is defined as:

$$\Delta\text{CoVaR}_q^{j|i} \;\; = \;\; \text{CoVaR}_q^{j|i} \; - \; \text{CoVaR}_{50\%}^{j|i} \tag{A.64}$$

where $\text{CoVaR}_{50\%}^{j|i}$ denotes the VaR of j's asset returns when i's returns are at their median (i.e. 50th percentile). In their paper, the authors focus on the case where $j = \text{system}$, i.e., when the return of the portfolio of all financial institutions is at its VaR level. In this case,

---

[52]A conceptually closely related model is the distressed insurance premium (DIP) of Huang, Zhou, and Zhu (2009b), which measures the conditional expected shortfall (CoES) of an institution, conditional on systemic distress. The DIP represents a hypothetical insurance premium against systemic distress, defined as total losses exceeding a threshold level of 15% of total bank liabilities. The implementation differs significantly from the CoVaR and Co-Risk models just described, however, and is based on *ex ante* default probabilities and forecast asset return correlations for individual banks, derived from market data including realized (high-frequency) correlations, interest rates, equity prices, and a volatility index.

the superscript j is dropped. Hence, ΔCoVaR^i denotes the difference between the VaR of the financial system conditional on the distress of a particular financial institution i and the VaR of the financial system conditional on the median state of the institution i. Thus, ΔCoVaR^i quantifies how much an institution adds to overall systemic risk.

Before discussing how to estimate CoVaR, we present how to estimate the market value of asset returns for financial institutions. Denote by ME_t^i the market value of institution i's total equity, and by LEV_t^i the ratio of total book assets to book equity. The growth rate of market valued total assets X_t^i is given by:

$$X_t^i \;\;=\;\; \frac{ME_t^i \cdot LEV_t^i \;-\; ME_{t-1}^i \cdot LEV_{t-1}^i}{ME_{t-1}^i \cdot LEV_{t-1}^i} \;\;=\;\; \frac{A_t^i - A_{t-1}^i}{A_{t-1}^i} \tag{A.65}$$

where $A_t^i = ME_t^i LEV_t^i$. Essentially, (A.65) is just applying the market-to-book equity ratio to transform book-valued assets into market valued assets.

To capture the time variation in the joint distribution of X^i and X^{system}, the conditional distribution is estimated as a function of state variables. The following two quantile regressions are run on weekly data:

$$X_t^i \;\;=\;\; \alpha^i \;+\; \gamma^i M_{t-1} \;+\; \epsilon_t^i \tag{A.66a}$$

$$X_t^{\text{system}} \;\;=\;\; \alpha^{\text{system}|i} \;+\; \beta^{\text{system}|i} X_t^i \;+\; \gamma^{\text{system}|i} M_{t-1} \;+\; \epsilon_t^{\text{system}|i} \tag{A.66b}$$

where M_t denotes a vector of state variables described in Table A.6. A quantile regression consists of optimizing a modified function shown below for the quantile regression in (A.66a):

$$\min_{\alpha_q,\beta_q,\gamma_q} \sum_t \begin{cases} q|X_t^i - \alpha_q - M_{t-1}\gamma_q| & \text{if } (X_t^i - \alpha_q - M_{t-1}\gamma_q) \geq 0 \\ (1-q)|X_t^i - \alpha_q - M_{t-1}\gamma_q| & \text{if } (X_t^i - \alpha_q - M_{t-1}\gamma_q) \leq 0 \end{cases}. \tag{A.67}$$

The quantile regression for (A.66b) is analogous. Having estimated the quantile regression parameters, the predicted values of VaR and CoVaR are:

$$\text{VaR}_t^i \;\;=\;\; \alpha^i \;+\; \gamma^i M_{t-1} \tag{A.68a}$$

$$\text{CoVaR}_t^i \;\;=\;\; \alpha^{\text{system}|i} \;+\; \beta^{\text{system}|i}\text{VaR}_t^i \;+\; \gamma^{\text{system}|i} M_{t-1} \,. \tag{A.68b}$$

Finally, ΔCoVaR_t^i for each institution is calculated as:

$$\begin{aligned} \Delta\text{CoVaR}_t^i(q) &= \text{CoVaR}_t^i(q) - \text{CoVaR}_t^i(50\%) \\ &= \beta^{\text{system}|i}(\text{VaR}_t^i(q) - \text{VaR}_t^i(50\%)) \,. \end{aligned} \tag{A.69}$$

Thus, in order to estimate institution i's contribution to systemic risk, ΔCoVaR_t^i, the quantile regressions in equation (A.66a) must be run twice: once for the desired q and once for $q = 0.5$.

Throughout the paper, the authors estimate ΔCoVaR_t^i for both the $q = 1\%$ and the $q = 5\%$ cases.

State Variables in M_t	Source
VIX	CBOE website
3M repo rate − 3M UST	Bloomberg (repo rate)
	FRBNY site (T-bill rate)
Weekly change of 3M T-bill rate	FRB H15 Release
Weekly Change in UST yield spread (10Y − 3M)	FRB H15 Release
Weekly Change in credit spread (10Y BAA bonds − 10Y UST)	FRB H15 Release
Weekly VW equity market return	CRSP
1Y real estate sector return via-	CRSP companies with-
the VW average return of real estate companies	SIC code 65–66

Table A.6: State variables used in CoVaR estimation.

E.1.2 Inputs and Outputs

The financial institutions on which the authors apply their CoVaR estimator are those in the CRSP database with two-digit SIC codes between 60 and 67 inclusive, indexed by PERMNO. Only firms with common shares are used.

- **Input:** Weekly data for the market value of equity for the institutions of interest for the period 1986:Q1–2010:Q1. This data is taken from CRSP.

- **Input:** Data on the book value of assets and book value of equity for each institution for the period 1986:Q1–2010:Q1. From this data, the book leverage (LVG) can be estimated. This data is available quarterly from the Compustat database. Since the book value data is quarterly, within a given quarter, the dynamics of X_t^i are driven solely by the dynamics of the market value of equity (see (A.65)).

  Note that X_t^{system} is simply the market capitalization weighted average of each institution's asset returns.

- **Input:** The weekly data described in Table A.6.

- **Input:** The desired quantile; the authors use both 1% and 5%.

- **Output:** The marginal contribution of institution i to systemic risk, $\Delta\text{CoVaR}_t^i(q)$ as defined in equation (A.69).

- **Output:** The VaR of institution j conditional on institution i being in distress. This can be found by running the quantile regressions system in (A.66) and evaluating (A.68) and (A.69) where *system* is replaced by j.

E.1.3 Empirical Findings

The authors find that, across institutions, there is only a very loose link between an institution's VaR and its contribution to systemic risk as measured by CoVaR^i. Hence, imposing financial regulation that is solely based on the individual risk of an institution in isolation might not be sufficient to insulate the financial sector against systemic risk.

The authors run regression analyses in order to examine what firm characteristics are good predictors of future CoVaR. They find that firms with higher leverage, more maturity mismatch, and larger size tend to be associated with larger systemic risk contributions one quarter, one year, and two years later, both at the 1% and the 5% levels.

They then use these firm variables to predict "forward ΔCoVaR" two years into the future, and find that it is negatively correlated with contemporaneous ΔCoVaR, i.e., the result of estimating it via the methods outlined in Section E.1.1. Thus, macroprudential regulation based on the "forward ΔCoVaR" is countercyclical.

For a detailed exposition of quantile regression techniques, see Koenker (2005) and for an intuitive exposition, see Koenker and Hallock (2001).

E.2 Distressed Insurance Premium

The Distressed Insurance Premium (DIP) is proposed as an *ex ante* systemic risk metric by Huang, Zhou, and Zhu (2009b) and it represents a hypothetical insurance premium against a systemic financial distress, defined as total losses that exceed a given threshold, say 15%, of total bank liabilities. The methodology is general and can apply to any pre-selected group of firms with publicly tradable equity and CDS contracts. Each institution's marginal contribution to systemic risk is a function of its size, probability of default (PoD), and asset correlation. The last two components need to be estimated from market data.

E.2.1 Definition

The PoD uses CDS quotes referring to 5-year, senior unsecured debt with no-restructuring clauses that are U.S.-dollar denominated. The authors use end-of-week observations to construct weekly CDS data. The 1-year risk-neutral PoD of the bank is given by:

$$\text{PoD}_{i,t} \quad = \quad \frac{a_t s_{i,t}}{a_t \text{LGD}_{i,t} + b_t s_{i,t}} \tag{A.70}$$

where $a_t = \int_t^{t+T} e^{-rx}\, dx$ and $b_t = \int_t^{t+T} x e^{-rx}\, dx$, LGD is the loss given default, $s_{i,t}$ is the CDS spread of bank i at time t, and r is the risk-free rate. The LGD is needed as an input: although one can use the Basel II recommendation of 55%, the authors choose to use the expected LGD as reported by market participants who price and trade CDS contracts.

The next step is to estimate the correlation of the banks' assets. The authors assume that at frequencies less than 12 weeks, leverage is approximately constant and thus asset correlation can be proxied by equity correlation. They test the constant leverage hypothesis and find that it holds for 10 of the 12 banks in their sample. Using high-frequency tick-by-tick data from the TAQ database, they estimate their measure of correlation as follows:

1. Since the market for the equity of the 12 banks is quite deep (more than one trade per second), they construct 30-minute intervals; for each stock, the last price observation in such a 30-minute interval is kept.

2. For each stock i, they compute the 30-minute geometric return for period j, denoted by $r_{(j),i}$, by taking the difference between the two adjacent logarithmic prices.

3. They define a horizon h for which the correlation is to be estimated (a week, a quarter, etc.) and then count the number M of 30-minute intervals in h. The pairwise correlation between stock k and l for this interval h is:

$$\hat{\rho}_{k,l} \;=\; \frac{\sum_{j=1}^{M} r_{(j),k} r_{(j),l}}{\sqrt{\sum_{j=1}^{M} r_{(j),k}^2 \sum_{j=1}^{M} r_{(j),l}^2}} \; . \tag{A.71}$$

4. Armed with the pairwise correlations over a given period, they compute the average correlation over that period denoted by $\rho_{t,t+T}$.

5. They use these average correlations as inputs to the following predictive regression:

$$\rho_{t,t+12} \;=\; c + k_1 \rho_{t-12,t} \;+\; \sum_{i=1}^{l} k_{2,i} \rho_{t-i,t-i+1} \;+\; \eta X_t \;+\; v_t \tag{A.72}$$

   where the subscript refers to the time horizon (one week as one unit) to calculate the correlations and X includes a list of financial market variables, which are the one-quarter return of the S&P 500, the current value of the VIX, the Fed Funds rate, and the 10-year 3-month treasuries spread.

6. The same authors propose a refinement in their correlation estimation procedure in a later paper, Huang, Zhou, and Zhu (2009b), by using the dynamic conditional correlation measure presented in Engle (2002). This method is useful if intraday bank equity data is not readily available.

Based on individual PoDs and forecasted asset return correlations, the indicator of systemic risk, which is the theoretical price of insurance against distressed losses in the banking sector over the next three months, can be calculated. As an example, "distress" can be defined as a situation in which at least 15% of total liabilities of the financial system are defaulted. To compute the indicator, they construct a hypothetical debt portfolio that consists of liabilities (deposits, debts, and others) of all banks in their sample, weighted by the liability size of each bank. The price of insurance against distress equals the expectation (under the risk-neutral world) of portfolio credit losses that equal or exceed the predetermined threshold. To estimate this expectation, they rely on the Monte Carlo method described in appendix B of Tarashev and Zhu (2008), which is summarized below.

The N banks' asset values are assumed to have common factors and idiosyncratic components as is the case in Vasicek's (1991) ASRF model of portfolio credit risk. A firm's

LGD is assumed to be independent of the factors underlying the stochastic process of its assets and the distribution of LGDs is identical across exposures. The simulation of portfolio credit losses can be divided into two parts: the first part calculates the probability distribution of joint defaults and the second part incorporates the LGD distribution to derive the probability distribution of portfolio losses.

Part 1:

1. Using the $N \times 1$ vector of PoDs and the assumption that asset returns are distributed as standard normal variables, they obtain an $N \times 1$ vector of default thresholds.

2. They draw an $N \times 1$ vector from N standard normal random variables which are all assumed to have a pairwise correlation equal to the forward looking average correlation measure estimated in (A.72). The number of entries in this vector, which are smaller than the corresponding default threshold, is the number of simulated defaults for the particular draw.

3. The above step is repeated 500,000 times to derive the probability distribution of the number of defaults, Pr(number of defaults $= k$), where $k = 0, 1, \ldots, N$.

Part 2:

1. For a given number of defaults k, they draw LGDs for the default exposure 1,000 times from a triangular distribution centered at 0.55 (Basel II recommendation) with range $[0.1, 1]$ and calculate the conditional loss distribution $Pr(TL|k \text{ defaults})$.

2. The above step is done for each $k = 1, \ldots, N$. Then, they are able to calculate the unconditional probability distribution of portfolio credit losses. Specifically:

$$\Pr(TL) = \sum_k \Pr(TL|k \text{ defaults}) \Pr(k \text{ defaults}) . \qquad (A.73)$$

3. Armed with the unconditional distribution of losses, they can then calculate the probability of the event that the total losses are above 15% of the total banking sector liabilities; to get the DIP, we then multiply the above probability with the expected losses conditional on the event that the losses are above 15% of the total banking sector liabilities.

E.2.2 Extensions

In Huang, Zhou, and Zhu (2009b), the authors propose two extensions of the DIP methodology used in Huang, Zhou, and Zhu (2009a) to analyze the sources of systemic risk: using objective probabilities of defaults instead of risk-neutral ones and identifying the systemic risk contribution of each institution.

For the former case, the systemic risk indicator uses PoDs that are estimated by using those supplied by Moody's KMV-EDF for each institution; these PoDs reflect the objective probability of default. The resulting DIP quantifies the contribution from the expected

actual defaults, and the difference between the market DIP (risk-neutral PoD case) and the actuarial DIP (objective PoD case) quantifies the contribution from the risk premia components.

For the latter case, the authors try to estimate the loss arising from a bank's default, conditional on the occurrence of the distress scenario where at least 15% of total liabilities are defaulted upon. Because systemic events are rare, the authors use importance sampling (see Glasserman and Li (2005)) to simulate these events. For the 22 bank portfolios in their sample, they generate 200,000 importance-sampling simulations of default scenarios (defaults or not), and for each scenario they generate 100 simulations of LGDs drawn from the distribution used in step 1 of part 2 of the simulation presented in Section E.2.1. Based on these simulation results, they estimate each bank's systemic risk contribution by calculating the expected loss of the bank conditional on total losses exceeding 15% of total liabilities.

E.2.3   Inputs and Outputs

- **Input:** To calculate the risk-neutral PoDs, daily 5-year CDS spread data for each bank are obtained from Markit; end-of-week observations are used to construct the weekly CDS dataset from January 2001 to December 2008.

- **Input:** The expected LGDs for each firm. The authors use expected LGDs as reported by market participants who price and trade CDS contracts. Alternatively, one can use the Basel II recommendation of 55%.

- **Input:** The objective PoDs are supplied by Moody's-KMV.

- **Input:** The raw high-frequency equity price data from the TAQ database to construct 30-minute interval prices in order to estimate pairwise correlations used in (A.72). Only the data during normal market hours (9:30am–4:00pm) are used.

- **Input:** Histories of the one-quarter return of the S&P 500 (from Bloomberg), the VIX (from Bloomberg), the Fed Funds rate (Fed's H15 Release), and the 10-year 3-month Treasuries spread (Fed's H15 Release) to use as inputs in estimating the forward looking average correlation in (A.72).

- **Output** The DIP systemic risk indicator resulting from the simulation presented in Section E.2.1.

E.2.4   Empirical Findings

The authors run the DIP indicator over time and find that the DIP per unit of overall liabilities started at 10 basis points in the first half of 2001, increased, and reached a peak of about 35 basis points in the second half of 2002, when high corporate defaults were reported. The indicator then trended downward and reached its lowest level in late 2006 and early 2007. Since August 2007, the indicator rose sharply, peaking around March 2008, and dropped dramatically after the Federal Reserve facilitated the acquisition of Bear Stearns by J.P. Morgan Chase. They also find that most of its movement was impacted by the changing PoDs although the correlation measure also plays a role.

In terms of the extensions presented in Huang, Zhou, and Zhu (2009b), the authors find that the actuarial DIP, i.e., using objective PoDs, is much smaller than the market DIP during the crisis. This suggests that during a crisis period, the bailout cost of a market-based solution is much larger than that justified by an objective assessment of the default losses due to increased risk aversion and decreased liquidity.

In terms of estimating banks' individual contributions to systemic risk, they find that typically, the largest banks coincide with the largest systemic risk contributors.

E.3 Co-Risk

The "Co-Risk" measure, first proposed in the IMF's 2009 *Global Financial Stability Review* (International Monetary Fund, 2009a), examines the co-dependence between the CDS of various financial institutions. It is more informative than unconditional risk measures because it provides a market assessment of the proportional increase in a firm's credit risk induced, directly and indirectly, from its links to another firm. Note that the approach used is based on quantile regression, a concept also used in CoVaR (see Adrian and Brunnermeier (2010), a summary of which is presented in Section E.1). The quantile regression approach permits a more accurate estimation of the co-movements of financial institutions' risk factors (or co-risk estimates) under distress, taking into account their nonlinear relationship.

E.3.1 Definition

The authors used daily five-year-maturity CDS spreads for a variety of financial institutions from July 1, 2003 to September 12, 2008. Intuitively, when an institution's CDS spreads are in their 5th quantile (the left tail of their distribution), this suggests that these institutions are experiencing an extremely benign regime, and when the CDS spreads are at their 95th quantile (the right tail of their distribution), this suggests a distress regime. Mathematically, the daily frequency quantile regression the authors run is:

$$\text{CDS}_{i,t} = \alpha_q^i + \sum_{m=1}^{K} \beta_{q,m}^i R_{m,t} + \beta_{q,j}^i \text{CDS}_{j,t} + \epsilon_{i,t} \tag{A.74}$$

where $\text{CDS}_{i,t}$ is the CDS spread of institution i on day t, $R_{m,t}$ is the value of risk factor m at time t, and q denotes the quantile. The parameter estimates, $\beta_{q,j}^i$, provide a measure of how firm j affects the credit risk of firm i (directly and indirectly) at different quantiles. Table A.7 presents the set of risk factors R_m that the authors use. Mathematically, the quantile regression consists of optimizing the function:

$$\min_{\alpha_q^i, \beta_{q,m}^i, \beta_{q,j}^i} \sum_t \rho_q \left(\text{CDS}_{i,t} - \alpha_q^i - \sum_{m=1}^{K} \beta_{q,m}^i R_{m,t} - \beta_{q,j}^i \text{CDS}_{j,t} \right) \tag{A.75}$$

where:

$$\rho_q(t) \quad = \quad \begin{cases} q|t| & \text{if } t \geq 0 \\ (1-q)|t| & \text{if } t < 0 \end{cases}. \tag{A.76}$$

After estimating the quantile regression coefficients, the conditional co-risk measure is defined as:

$$\text{CoRisk}_t^{i,j} \quad = \quad 100 \times \left(\frac{\alpha_{95}^i + \sum_{m=1}^K \beta_{95,m}^i R_{m,t} + \beta_{95,j}^i \text{CDS}_j(95)}{\text{CDS}_i(95)} - 1 \right) \tag{A.77}$$

where $\text{CDS}_i(95)$ is the CDS spread of institution i corresponding to the 95$^{\text{th}}$ percentile of its empirical sample and the alphas and betas are estimated running a quantile regression with $q = 0.95$. A higher co-risk measure indicates increased sensitivity of institution i's credit to distress in institution j's credit.

Risk Factor	Source
VIX	CBOE website
3M repo rate − 3M UST	Bloomberg (repo rate)
	FRBNY site (T-bill rate)
S&P 500 return − 3M UST	CRSP (S&P), FRB H15 (3M UST)
Yield curve slope (10Y − 3M)	FRB H15 Release
LIBOR spread	Bloomberg (LIBOR)
(1Y LIBOR − 1Y constant maturity UST yield)	FRB H15 Release (UST yield)

Table A.7: Daily risk factors used in co-risk estimation.

E.3.2 Inputs and Outputs

- **Input:** Daily five-year-maturity CDS spreads for a variety of financial institutions from July 1, 2003 to September 12, 2008. CDS mid-price quotes were obtained from Bloomberg and Primark Datastream.

  The institutions the authors studied are AIG, Bank of America, Bear Stearns, Citigroup, Goldman Sachs, J.P. Morgan, Lehman Brothers, Merrill Lynch, Morgan Stanley, Wachovia, Wells Fargo, Fortis, BNP, Société Générale, Deutsche Bank, Commerzbank, BBVA, Banco Santander, Credit Suisse, UBS, Barclays, HSBC, Mitsubishi, Mizuho, and Sumitomo.

- **Input:** The daily data described in Table A.7.

- **Input:** The desired quantile; the authors use $q = 95\%$.

- **Output:** The co-risk measure presented in equation (A.77).

E.3.3 Empirical Findings

The authors find that in March 2008, when Citigroup's CDS spreads were at their 95th percentile, this would have led to an increase of 135% in Bear Stearns' CDS spread and 103% in Lehman's CDS spread. Furthermore, the risk of Bear Stearns conditional on the risk of AIG is 248% higher than that corresponding to the 95th percentile of Bear Stearns' empirical distribution. Thus, these results suggest that AIG, Bear Stearns, and Lehman, should have been closely monitored in March 2008.

The authors also find that in March 2008, the conditional co-risk of AIG and Lehman to the rest of the institutions in the sample were, on average, 11% and 24%, respectively. However, on September 12, 2008, these estimates jumped to 30% and 36%, respectively, thus highlighting the increased systemic importance of these institutions at the peak of the crisis.

A more detailed exposition of the co-risk measure can be found in Chan-Lau (2009). For a detailed exposition of quantile regression techniques, see Koenker (2005) and for an intuitive exposition, see Koenker and Hallock (2001).

E.4 Marginal and Systemic Expected Shortfall

Acharya, Pedersen, Philippon, and Richardson (2010) argue that each financial institution's contribution to systemic risk can be measured as its systemic expected shortfall (SES), i.e., its propensity to be undercapitalized when the system as a whole is undercapitalized. SES is a theoretical construct and the authors use the following 3 measures to proxy it:

1. The outcome of stress tests performed by regulators. The SES metric of a firm here is defined as the recommended capital that it was required to raise as a result of the stress test in February 2009.

2. The decline in equity valuations of large financial firms during the crisis, as measured by their cumulative equity return from July 2007 to December 2008.

3. The widening of the credit default swap spreads of large financial firms as measured by their cumulative CDS spread increases from July 2007 to December 2008.

Given these proxies, the authors seek to develop leading indicators which "predict" an institution's SES; these leading indicators are marginal expected shortfall (MES) and leverage (LVG). They provide a micro foundation argument about the theoretical relationship of these two indicators to SES, which we will not discuss here. The authors test MES and LVG with respect to their predictive power for the three SES proxies above and find decent predictability.

E.4.1 Definition

A firm's MES is defined as the average return of its equity (R_b) during the 5% worst days for the overall market return (R_m), where the market is proxied by the CRSP Value Weighted Index:

$$\text{MES}_b \quad = \quad \frac{1}{\text{number of days}} \sum_{\{t:\ \text{system is in its 5\% tail}\}} R_{bt} \ . \qquad (A.78)$$

114

Due to the fact that it is not straightforward to measure true leverage because of limited and infrequent market data, especially on the breakdown of off- and on-balance sheet financing, the authors apply the standard approximation of leverage:

$$
\begin{aligned}
\mathrm{LVG}_b &= \frac{\text{quasi-market value of assets}}{\text{market value of equity}} \\
&= \frac{\text{book assets} - \text{book equity} + \text{market equity}}{\text{market value of equity}} \,.
\end{aligned} \tag{A.79}
$$

The authors run cross-sectional regression analyses of firms' SES on MES and LVG:

$$
\mathrm{SES}_i = a + b\mathrm{MES}_i + c\mathrm{LVG}_i + \epsilon_i \,. \tag{A.80}
$$

In terms of generating a systemic risk indicator, (a, b, c) are estimated for a specific metric of SES. For example, for the second metric of SES, for each firm, the authors use the returns data from June 2006 to June 2007 to estimate the corresponding MES and they use the appropriate balance sheet data from June 2007 to estimate the corresponding LVG. They then use the cumulative equity return for each firm from July 2007 to December 2008 to get their proxy of each firm's SES. After running the cross-sectional regression in (A.80) to estimate the triplet (a, b, c), the systemic risk that firm i poses at a future time t is calculated as:

$$
\text{Systemic Risk of Firm } i = \frac{\hat{b}}{\hat{b} + \hat{c}}\mathrm{MES}_i^t + \frac{\hat{c}}{\hat{b} + \hat{c}}\mathrm{LVG}_i^t \tag{A.81}
$$

E.4.2 Variants

The first variant is the F-MES: instead of using the 5% worst days of the overall market in defining the days which the firm's returns are averaged, they use the 5% worst return days of the financial subsector. The results using the F-MES are virtually identical to the original MES described above.

They also define another measure of MES that uses CDS data as the primitive. They use the CDS of the 40 financial firms which have CDS traded on their debt. They construct the CDS MES for a firm by taking the 5% worst days for an equally weighted portfolio of the 40 CDS returns and then compute the firm's average CDS return for these days. They find predictability of SES using this measure as well.

E.4.3 Inputs and Outputs

- **Input:** Daily returns for the CRSP Value-Weighted Index and the financial firms of interest obtained from CRSP from June 2006 to June 2007 to calculate MES.

- **Input:** Daily CDS spread data for the financial firms of interest from June 2006 to June 2007 obtained from Markit. This input is used for the CDS MES variant presented in Section E.4.2.

- **Input:** Market value of each firm's equity, book value of assets, and book value of equity obtained from the merged CRSP/Compustat database for June 2007 to calculate quasi-leverage.

- **Output:** The systemic risk contribution of firm i is defined as in (A.81).

E.4.4   Empirical Findings

Firstly, it should be noted that the authors run detailed regression analyses to assess the predictability of MES and LVG on each of the three metrics of SES. Overall, they find relatively high R-squared values ranging between 20% and 60%. An interesting finding is that insurance firms are deemed the least systemically risky; this is in contradiction to other recent empirical findings (see Billio, Getmansky, Lo, and Pelizzon (2010) and Adrian and Brunnermeier (2010)) as well as the theoretical role of the insurance industry in the financial markets (see Sapra (2008)); furthermore, AIG was clearly a systemic risk in the recent financial crisis. They also find that securities broker/dealers are the riskiest, presumably because they are the most levered institutions.

In exploring the robustness of MES as a predictor, they find that its predictive power progressively declines as they increase the lag in the data used to compute it, suggesting that using the most up-to-date data is optimal in terms of the usefulness of the MES as a systemic risk indicator input.

F   Measures of Illiquidity and Insolvency

A number of risk metrics focus on the status of a financial institution or system at a point in time—how a firm's or a financial system's contracts expose it to risks—but systemic risk measurement is also concerned with dynamic behavior, especially during a crisis. It is therefore important to watch those aspects of the system that can clarify how behavior might change in response to a significant shock. For example, responses to stress are the focus of Brunnermeier, Gorton, and Krishnamurthy (2010)'s "risk topography" proposal described below. All economic behavior plays out subject to constraints, and significant financial constraints are likely to bind during a stress episode. These limitations on behavior can form the basis for an approximate *ex ante* indication of participants' responses.

Access to funding sources constrains an institution's portfolio adjustment opportunities in the short run. For example, most banks perform maturity transformation—borrowing short-term while committing to longer-term assets—as an integral part of their financial strategy, thus exposing them to liquidity risk. In a stress episode, an action available to lenders in short-term funding markets is to disintermediate: demand deposits can be withdrawn, and wholesale lenders (e.g., overnight Fed funds and repos) can refuse to roll over their placements. In addition, funders of a dealer bank can attempt to neutralize their exposures by drawing on lines of credit from that dealer, or by engaging in new derivatives positions with the dealer to offset existing contracts. An institution caught in a liquidity squeeze might draw on other funding sources such as backup credit lines, Federal Home Loan Bank advances, or the Federal Reserve's discount window. Traditional duration gap and repricing gap measures can capture these exposures at the level of the individual institution, and can also be aggregated across firms. Alternatively, a stressed institution can sell liquid assets

to raise cash. Small sales in active markets are unlikely to create large price movements: a seller would ordinarily work off a large position gradually to allow time for new bidders to arrive and replenish the rolls. In a crisis such patience is a luxury, and asset sales may eat deeply into the set of buyers available immediately. Compounding the problem, potential buyers will exploit their bargaining power via lower offering prices if they detect the seller is liquidating under duress.

Leverage—the ratio of a position's economic value to the size of the capital cushion supporting it—is an important feature of financial crises. Leverage simultaneously magnifies the percentage returns to capital investors and increases the risk of insolvency. Leverage is also connected to liquidity and maturity through the financial structure of most intermediaries. A commercial bank, for example, invests in a diversified portfolio of illiquid, relatively long-term loans, and issues liquid, short-term deposits. The law of large numbers creates the predictability across both depositor withdrawals and loan defaults needed to make this arrangement workable. McCulley (2010) emphasizes that "*ex ante* liquidity"—the notion that a depositor could withdraw at will—is in relatively high demand, reducing the yield banks must pay on their demandable liabilities. This is a game of trust, because "ex-post liquidity" is not supportable on the same scale; a requirement to pay out depositor withdrawals in actuality will cause insolvency. To limit the externalities that come with failure, banks have recourse to deposit insurance and the central bank's last-resort lending services to help insure the stability of this arrangement. However, this generates a moral hazard by insulating banks from their own liquidity risk, creating a need for prudential regulation. "Shadow" banks, on the other hand, also perform liquidity and maturity transformations, but position themselves to benefit from taxpayer-supported liquidity facilities without subjecting themselves to the countermanding regulatory regime.[53] Financial Stability Board (2011) emphasizes that this can be profitable, but also destabilizing.

One way for banks (shadow or otherwise) to benefit from this moral hazard is through excessive leverage, which magnifies equity-holders' positive returns, while imposing on taxpayers the losses in the event of bank failure. As a policy matter, it is insufficient to consider leverage only on a firm-by-firm basis: coordinated bank failures impose greater external costs than a single bank failure in isolation. Nijskens and Wagner (2011), for example, examine the risk profiles of banks over the 1997–2006 period, finding that the securitization of commercial loans through collateralized loan obligations (CLOs) that reduced idiosyncratic bank risk simultaneously increased systemic risk by increasing betas and return correlations across institutions. Acharya, Pedersen, Philippon, and Richardson (2010) consider this systemic dimension with a collection of metrics that extend the familiar institution-level VaR and expected shortfall (ES) notion to capture an institution's "systemic expected shortfall" (SES): its propensity to be undercapitalized when the system as a whole is undercapitalized. They also devise marginal expected shortfall (MES) and leverage metrics as early warning indicators of SES.

SES is a static measure, in the sense that it represents a measurable risk exposure of the institution at a particular point in time. Minsky (1982), in contrast, focuses on the endogenous dynamics of leverage. The crux of Minsky's instability hypothesis is the tendency of participants to increase their leverage over time. During episodes of rising asset prices, both speculation and increasing leverage are profitable *ex post*. (In a price bubble, this profitabil-

---

[53]Pozsar, Adrian, Ashcraft, and Boesky (2010) describe the shadow banking system in detail.

ity becomes self-fulfilling). Geanakoplos (2010) emphasizes the role of the marginal investor: above (below) her along the demand schedule are other buyers who are more (less) enthusiastic about the security. Increasing leverage means that a smaller set of (more enthusiastic) investors can determine the price. In a boom, leverage and enthusiasm thus feed on each other; the process is unsustainable if left unchecked. Eventually, the dynamic is reversed—perhaps by a random shock that imposes magnified losses on over-levered investors—and a self-reinforcing episode of price declines and deleveraging ensues. In short, the range of possible portfolio adjustments—which assets can be sold at what prices—is likely to change markedly during a stress episode. While some changes will only be apparent after the fact, other indicators—for example, based on bid-ask spreads, public limit order books, transaction volumes, and the characteristics of market-makers and other liquidity providers—can provide *ex ante* evidence of deterioration in market quality. Hu, Pan, and Wang (2010) document this in the context of the market for U.S. Treasury securities. They derive a measure of market noise as the deviation of observed market yields on Treasury bonds from their model-based yields derived from a daily estimate of the zero-coupon curve. They conclude that noise is typically quite low (and liquidity correspondingly high), enforced by arbitrage; however, noise spikes during crises as arbitrage capital exits the marketplace. Moreover, in a systemic crisis, many participants are likely to be liquidating simultaneously, further exacerbating short-term pressure on prices, while liquidity providers with thin capitalization (e.g., high-frequency traders) may exit the market altogether. Such coordinated behavior is the essence of Pojarliev and Levich (2011)'s study of "crowded trades", which uses a proprietary high-frequency dataset of currency funds' returns to examine the implications of their trading styles on price and correlation dynamics as well as the funds' own profitability.[54]

Similar concerns arise in the other markets. Khandani and Lo (2011) propose two distinct measures of equity market liquidity. They consider a contrarian trading strategy of buying losers and selling winners. This is essentially a market-making strategy that smooths temporary supply-demand imbalances and compensates the trader for providing liquidity. Its profitability has been generally decreasing since the late 1990s, as increasing competition in this strategy has reduced liquidity premiums. Their first measure examines the extent of high-frequency mean-reversion in transacted prices. Their second metric is "Kyle's lambda", originally due to Kyle (1985), which measures liquidity via a regression estimate of the volume required to cause a one-dollar price movement.

Getmansky, Lo, and Makarov (2004) explore the sources of serial correlation in *reported* hedge-fund returns, and conclude that the most likely explanation is illiquidity exposure and smoothed returns. By definition, current prices in illiquid markets are frequently unavailable or unreliable, forcing funds to report mark-to-model estimates that often rely linear extrapolation pricing methods. Serial correlation in observed returns is an artifact of this autoregressive smoothing, and it thus offers a proxy measure for illiquidity.[55]

Counterparty insolvency also affects the set of portfolio-adjustment possibilities. Typi-

---

[54]They define four "style factors" as proxy measures for trading strategies and assess crowdedness as the net proportion of funds in their sample exposed to each factor. They detect a significant connection between crowdedness and performance for currency funds.

[55]Chan, Getmansky, Haas, and Lo (2006b, 2006b) propose a related autocorrelation measure. They also estimate liquidation probabilities for hedge funds via logit regressions on samples of "live" and non-reporting funds, and a regime-switching model (between high- and low-return states) for a variety of hedge-fund indices that can identify episodes of coordinated downturns.

cally, counterparties with short exposures to a defaulting firm do not lose everything on their transactions. For example, bilateral and multilateral netting agreements can sharply reduce losses given default, and many contracts include recourse to collateral in the event of counterparty default. Participation of individual contracts in netting or collateral arrangements is in principle an observable fact. If collateral provisions exist, it is useful to know whether collateral is pledged under a blanket lien or a specific lien, and what the collateral haircuts are. Similarly, for positions collateralized through margining agreements, it is valuable to monitor the margin requirements, including procedures for calculating variation margin and marking accounts to market. Haircuts and margins can change over time, and observation of these evolving values can provide valuable insights into creditworthiness. This is a central element of Brunnermeier, Gorton, and Krishnamurthy (2010)'s proposal to monitor systemic risk via a two-step "risk topography" process. In the first step, regulators would accumulate a panel data set of participants' individual risk exposures (changes in firm value) as well as liquidity sensitivities (changes in "effective cash") to various shock scenarios defined on a space of external factors. A key innovation is the introduction of a summable and state-contingent liquidity index for portfolio positions, to capture changes in fire-sale discounts or in collateral haircuts. The second step would aggregate firms' individual valuation and liquidity responses into a system-wide picture of risks, where those exposures diversified in the cross-section of firms should be of less concern that systemically concentrated exposures. Lastly, many positions are protected with third-party guarantors or in the CDS markets. In these cases, it is important to know the nature and amount of the protection, and to have a sense of the creditworthiness of the guarantor: wrong-way risk occurs when exposure to a counterparty correlates adversely with counterparty creditworthiness. For example, many MBS bondholders discovered that the concentrated incidence of claims during the mortgage crisis rendered guarantees from monoline subprime insurers unlikely to pay out precisely when the insured event occurred. If a guarantor's or counterparty's other exposures are observable, then estimates of wrong-way risk become possible.

Liquidity and short-run portfolio flexibility present measurement challenges in many ways. For example, many markets do not publicly broadcast all the bids and offers—the effective market depth—so that empirical studies have resorted to a variety of work-arounds. Hu, Pan, and Wang (2010) finesse the problem by tracking pricing "noise" as a signal of the absence from the market of arbitrage capital. Pojarliev and Levich (2011) assign hedge funds to style categories to proxy for crowdedness in trading strategies. Khandani and Lo (2011) use market trading volume to help filter out the Kyle's lambda metric for liquidity. Getmansky, Lo, and Makarov (2004) and Chan, Getmansky, Haas, and Lo (2006a) use autocorrelation in reported hedge-fund returns to suggest which strategies are forced to use linear price extrapolations rather than observed transaction prices to mark their portfolios. Notably, Brunnermeier, Gorton, and Krishnamurthy (2010) propose to go much further, measuring not only a full panoply of external market risk factors (interest rates, credit spreads, currency and commodity prices, etc.), but also direct assessment of repo haircuts and the availability of funding and loan syndication markets; it is also an approach that as yet lacks an empirical implementation. Ideally, measurement of liquidity and portfolio flexibility would have access to a diverse array of liquidity information, including bid-ask spreads, limit-order depths, transaction volumes and order flows (where available), and the details of netting and collateral/margin agreements.

F.1 Risk Topography

Brunnermeier, Gorton, and Krishnamurthy (2010) propose to create a "risk topography" of the financial system involving a data acquisition and dissemination process that informs policymakers, researchers, and market participants about systemic risk. Specifically, they propose a two-step process: first, regulators elicit from market participants their (partial equilibrium) risk as well as liquidity sensitivities with respect to major risk factors and liquidity scenarios. By doing so, one takes advantage of private sector internal risk models and over time an informative panel dataset is obtained. Second, general equilibrium responses and economy-wide system effects are calibrated using this panel dataset.

To gauge the liquidity, it is not enough to simply look at current asset market measures like current volatility, liquidity, spreads etc. Rather it is important to: (i) measure and model how market participants endogenously respond (in a partial equilibrium sense) to negative shocks; and (ii) determine how feedback responses produce system-wide or aggregate general-equilibrium dislocations.

In addition to reporting risk and liquidity sensitivities, each firm reports a vector in the factor space that points in the direction of factor movements to which the firm has the highest exposure. If all firms' reported vectors point in a similar direction, this particular cross-scenario has to be added in next quarter's survey. The information should be kept simple and straightforward, and then continued on a regular basis such that over time, a rich panel of data will accumulate.

To illustrate the key points, the authors present a simple one-period model: there are two dates, a date 0 *ex ante* period in which assets and liabilities are chosen, and a date 1 with states $\omega \in \Omega$ when we may face a systemic crisis. At time 0, firm i chooses assets of A_i and liabilities of L_i. The assets A_i are a mix of cash, repo lending to other firms, derivative exposure, and outright asset purchase. Liabilities include short-term debt, long-term debt, secured debt, equity, etc.

The equity value of the firm at state ω is given by $E_\omega^i = A_\omega^i - L_\omega^i$. In addition, we are interested in the liquidity position of each firm. For tractability, each asset/liability is assigned a liquidity index λ_ω^j for each state of the world. Assets have positive λs and liabilities have negative λs. Super-liquid monetary assets such as bank reserves and Treasuries have $\lambda = 1$ across all states; overnight debt has a liquidity of -1. For something like a mortgage-backed security (MBS), we can imagine measuring λ_ω^{MBS} as one minus the repo haircut on that MBS in state ω. Alternatively λ_ω^{MBS} may measure the price discount that firm has to accept if it immediately wanted to convert the asset into cash. Aggregating liquidity across the asset side, one obtains firm i's market liquidity $\Lambda_\omega^{A,i}$ for the different states of the economy. Aggregating liquidity across the liability side, one obtains firm i's funding liquidity $\Lambda_\omega^{L,i}$ for the different states of the economy. Common equity has a λ of 0.

Factors consist of certain prices (risk factors) or liquidity/funding conditions (liquidity factors). For example a risk factor might be a change in real estate prices, while a liquidity factor could be a change in haircuts and margins. Along these dimensions, firms report a "value-liquidity" vector that consists of their estimated: (i) total asset value; (ii) equity value; (iii) asset liquidity index ($\Lambda_\omega^{A,i}$); and (iv) funding liquidity index ($\Lambda_\omega^{L,i}$). For example, if there is only one risk factor e.g., with N real estate price levels, and one liquidity factor, e.g., with M overall haircut levels, then the state space can be characterized by a $N \times M$ matrix. Firms have to report their estimated values-liquidity indexes for each combination. From

this, one can derive the partial sensitivities of each firm along each single factor. Finally, firms report the four states in which they would experience the four highest losses in value and liquidity index, respectively. As long as all firms pick different (n, m) combinations, systemic risk seems contained. However, when many firms start picking similar (n, m) combinations, this points to a systemic risk concentration.

Given the above responses, one can estimate the change in each of the 4 "value-liquidity" elements above with respect to a change in a risk factor, a liquidity factor, or combinations of both; denote this sensitivity matrix by Δ_i. If the data collection exercise is repeated every quarter for a number of years for many firms for some core set of states, then over time, we will have data on the risk exposure choices, Δ_i, made by firm i for different value-liquidity vectors for firm i and in varying macro-state market conditions. As this panel accumulates, the data can be used to build and test models of how firm i makes its portfolio choice decision.

Examples of risk scenarios include changes in asset prices (equities, commodities, etc.). Examples of liquidity scenarios include rising haircuts, inability to access cash market over some time, etc. Examples of other types of factors are default of a counterparty, rating downgrade of the firm, etc.

The responses of the firms can also be aggregated to measure net exposures to various risk factors. For example, denote by Δ_{A_i} the change of firm i's total assets with respect to a change in the real estate risk factor. Then one can aggregate across firms:

$$\sum_{i=1}^{I} \Delta_{A_i} \tag{A.82}$$

where I is the total number of firms reporting. This sum is the total exposure of all measured firms to real estate risk. We would expect that some firms are long exposure and others are short exposure. At less aggregated levels, say sectorally, the risk aggregates are likely to be informative as they will reveal pockets of risk concentration and can serve to diagnose systemic risk.

The liquidity measures can also be aggregated. The net liquidity index for firm i is $\Lambda^i = \Lambda^{A,i} - \Lambda^{L,i}$. Then, summing over all firms:

$$\sum_{i=1}^{I} \Lambda^i . \tag{A.83}$$

Summed across all sectors, the liquidity aggregate should equal the supply of liquid assets: the λ-weighted sum across all relevant assets. The aggregates are more interesting in describing the liquidity position of particular sectors. For example, the banking sector probably carries a negative liquidity position, while the corporate sector carries a long liquidity position.

F.2   The Leverage Cycle

Geanakoplos's (2010) "leverage cycle" refers to the following phenomenon: there are times when leverage is so high that people and institutions can buy many assets with very little money down and times when leverage is so low that buyers must have all or nearly all of the money in hand to purchase those very same assets. When leverage is loose, asset prices go up because buyers can get easy credit. Similarly, when leverage is highly constrained, prices plummet. Governments have long monitored and adjusted interest rates in an attempt to ensure that credit did not freeze up and thereby threaten the economic stability of a nation. However, leverage must also be monitored and adjusted in order to avoid the destruction that the tail end of an outsized leverage cycle can bring.

The author points out that the collateral rate (or leverage), is an equilibrium variable distinct from the interest rate. Huge moves in collateral rates are deemed "leverage cycles" and are a recurring phenomenon in American financial history. These cycles typically end with:

1. Bad news that creates uncertainty and disagreement;
2. Sharply increasing collateral rates;
3. Losses and bankruptcies among the leveraged optimists.

The above factors reinforce and feed back on each other. Once the crisis has started, the solution is to reverse the three symptoms of the crisis: contain the bad news, intervene to bring down margins, and carefully inject "optimistic" equity back into the system. The author thus advocates a permanent lending facility that will stand ready, should another crisis arise, to give loans with less collateral than the market demands. Furthermore, he suggest principal reduction by private lenders. Moreover, he suggests injecting equity to financial institutions as a way to prop up the "leveraged optimists".

With respect to the current crisis and its relation to the leverage cycle, the author finds that the story line from the early- to mid-2000s to today fits very well with his theory as housing prices rose in lockstep with homeowner leverage during that period. He mentions that during this cycle, securities leverage got higher than ever before due to financial innovations in collateral; however, as the bubble burst, leverage also fell more than ever before. He makes an interesting point that the appearance of CDSs on mortgage bonds as a way to bet against the housing market allowed pessimists to leverage their negative views and thus actively push prices down. A small initial drop in housing prices led to some delinquencies and pessimistic views were expressed via the CDS markets. In fact, the collapse in mortgage bond prices preceded the more severe housing price decline later on. According to the author, if these CDSs had existed from the beginning, house prices may not have gotten so high in the first place; however, their appearance near the peak of the housing bubble guaranteed that there would be a large drop in housing prices.

In terms of managing the cycle during its ebullient phase, the author gives a list of recommendations so that the next cycle does not reach such a devastating crisis stage:

1. The Fed must collect data from a broad spectrum of investors, including hitherto secretive hedge funds, on how much leverage is being used to buy various classes of assets. Moreover, the amount of leverage being employed must be transparent. For example, current haircuts on assets should be monitored and reported.

2. The Fed should impose outside limits on leverage at the individual security level.

3. Tax firms that borrow excessively, or that borrow excessively on their collateral, or that lend excessively on collateral. A very small tax might go a long way to discourage excessive leverage, and might also change the maturity structure, inducing longer term loans, if it were designed properly. Another advantage of the leverage tax is that revenues from it could be used to finance the lending facility the Fed would need to keep at the ready in anticipation of the downside of future leverage cycles.

4. Mandate that lenders can only tighten their security margins very slowly. Knowing they cannot immediately adapt if conditions get more dangerous, lenders will be led to keep tighter margins in good, safe times.

F.3 Noise as Information for Illiquidity

Hu, Pan, and Wang (2010) consider the amount of arbitrage capital in the market and the potential impact on price deviations in the market for U.S. Treasury securities. During market crises, the shortage of arbitrage capital leaves the yields to move more freely relative to the curve, resulting in more "noise". As such, noise in the Treasury market can be informative about liquidity in the broad market because of the central importance of the Treasury market and its low intrinsic noise, i.e., high liquidity and low credit risk.

F.3.1 Definition

Using the CRSP Daily Treasury database, the authors construct their noise measure by first backing out, day by day, a smooth zero-coupon yield curve; in doing so, the authors use all bonds available on a given day with maturities between 1 month and 10 years. This zero-coupon curve is then used to price all available bonds on that day. Associated with each bond is the deviation of its market yield from the model yield. Aggregating the deviations across all bonds by calculating the root mean squared error, they obtain their noise measure.

In order to back out a day's smooth zero-coupon yield curve, the authors rely on the Svennson (1994) model, which assumes that the instantaneous forward rate f is given by:

$$f(m,b) \;=\; \beta_0 \;+\; \beta_1 \exp(-\frac{m}{\tau_1}) \;+\; \beta_2 \frac{m}{\tau_1} \exp(-\frac{m}{\tau_1}) \;+\; \beta_3 \frac{m}{\tau_2} \exp(-\frac{m}{\tau_2}) \tag{A.84}$$

where m denotes the time to maturity, and $b = (\beta_1, \beta_2, \beta_3, \tau_1, \tau_2)$ are model parameters to be estimated. Using the parameterized forward curve, the zero-coupon yield curve can be derived by:

$$s(m,b) \;=\; \frac{1}{m} \int_0^m f(m,b)dm \;. \tag{A.85}$$

Thus, by observing the coupon bonds on a given day, one can back out the zero-coupon rate s for each maturity. Let N_t be the number of bonds with maturities between 1 month and 10 years available on day t for curve-fitting and let P_t^i, $i = 1, \ldots, N_t$, be their respective

market observed prices. The model parameters b_t are chosen by minimizing the sum of the squared deviations between the actual prices and the model-implied prices:

$$b_t = \arg\min_{b_t} \sum_{i=1}^{N_t} \left[P^i(b_t) - P_t^i \right]^2 \tag{A.86}$$

where $P^i(b_t)$ is the model-implied price for bond i on day t given the model parameters b_t. Thus, on each day t, the end product of the curve-fitting is the vector of model parameters b_t.

In constructing the "noise" measure for a particular day, the authors use all bonds with maturities between 1 and 10 years available on that day. The "noise" measure does not use bonds with maturities less than 1 year because their information content may be limited due to the fact that the short end is typically not the object of arbitrage capital; thus, the short end is used for calibration purposes only.

Let n_t denote the number of bonds available on day t, and y_t^i bond i's observed market yield. Then, Noise$_t$ is defined as:

$$\text{Noise}_t = \sqrt{\frac{1}{n_t} \sum_{i=1}^{n_t} [y_t^i - y^i(b_t)]^2} \ . \tag{A.87}$$

F.3.2 Inputs and Outputs

- **Input:** Daily bond prices and yields obtained from the CRSP Daily Treasury database from January 1987 to December 2009.

- **Output:** The daily liquidity "noise" measure as defined in (A.87).

F.3.3 Empirical Findings

The time series of the liquidity noise measure reveals that during normal times, the noise measure is highly persistent and small, meaning that the arbitrage capital on the yield curve is effective in keeping the deviations within a range that is unattractive given the transaction cost. During crises, however, the noise measures spike up, implying a high degree of misalignment in the yield curve that would have been attractive for relative value trading during normal times and are in fact attractive given the contemporaneous transaction cost. Furthermore, the authors find a positive relationship between the noise measure and the on-the-run premium; while the noise measure is, on average, smaller than the on-the-run premium, it tends to spike up much more significantly during crises. The fact that the noise measure spike much more prominently than the on-the-run premiums during liquidity crises implies that there is commonality in the pricing errors across the entire yield curve. And the heightened commonality during crises is reflected in noisy and misaligned yield curves, which are captured by the noise measure.

The authors also find that factors known to be related to liquidity, such as the VIX, the slope of the yield curve, the Pastor and Stambaugh (2003) liquidity factor, and the RefCorp

spread (see Longstaff (2004)), have a significant relation with their proposed liquidity noise measure.

The authors also find that their liquidity noise measure can help explain the cross-sectional variation in hedge fund returns and currency carry trades, both of which are thought to be sensitive to market liquidity conditions.

F.4   Crowded Trades in Currency Funds

Pojarliev and Levich (2011) propose a method for detecting "crowded trades" in the currency fund world, but their approach may be used to measure the popularity or crowdedness of any trade with an identifiable time-series return. They focus on currency funds because they have access to a new database of daily currency fund data. The high-frequency data in their sample allowed them to develop measures of crowdedness over economically relevant horizons. They define style crowdedness as the percentage of funds with significant positive exposure to a given style less the percentage of funds with significant negative exposure to the same style. To estimate crowdedness, they use data from 107 currency managers that cover the period from April 2005 to June 2010.

F.4.1   Definition

The authors estimate style betas by using the four-factor model proposed in Pojarliev and Levich (2008):

$$R_t \;=\; \alpha \;+\; \Sigma_i \beta_i F_{i,t} \;+\; \epsilon_t \tag{A.88}$$

where:

$$
\begin{aligned}
R_t &= \text{The excess weekly return generated by the currency manager} \\
F_{i,t} &= \text{Factor } i \text{ value at time } t \\
\beta_i &= \text{Loading on factor } i
\end{aligned}
$$

The four risk factors that the authors use are:

1. **Carry Factor:** The Deutsche Bank Currency Harvest G10 Index is used as the proxy for the returns of a carry strategy. This index reflects the return of being long the three high-yielding currencies against being short the three low-yielding currencies among the G10 currencies. The Bloomberg code for this factor is DBHVG10U.

2. **Trend Factor:** The AFX Currency Management Index is used as a proxy for the trend-following factor. The AFX Index is based on trading in seven currency pairs weighted by their volume of turnover in the spot market, with returns for each pair based on an equally weighted portfolio of three moving average rules (32, 61, and 117 days).

3. **Value Factor:** The Deutsche Bank G10 Valuation Index is used as the proxy for the returns of a value strategy. This index reflects the return of being long the three most

undervalued currencies against being short the three most overvalued currencies among the G-10 currencies. The Bloomberg code for this factor is DBPPPUSF.

4. **Currency Volatility Factor:** The first difference of the (weekly) Deutsche Bank Currency Volatility Index is used as the proxy for foreign exchange volatility. The Bloomberg code for this factor is CVIX.

The authors propose two similar definitions of crowdedness for a given factor:

1. **Definition 1:** The crowdedness of factor F_i at time t $(C_{F_i,t})$ is defined as the percentage of funds with statistically significant positive exposure to factor F_i less the percentage of funds with statistically significant negative exposure to the same factor:

$$C_{F_i,t} = a_{F_i,t} - b_{F_i,t} . \qquad (A.89)$$

Here $a_{F_i,t}$ is the percentage of funds with statistically significant positive exposure to risk factor F_i over the period $t - 25$ weeks through t, and $b_{F_i,t}$ is the percentage of funds with statistically significant negative exposure to risk factor F_i over the period $t - 25$ weeks through t. For both negative and positive exposures, the authors used a standard 95% confidence interval, implying a t-statistic above 1.96 in absolute value.

2. **Definition 2:** The crowdedness of factor F_i at time t, $(C^*_{F_i,t})$ is defined as the percentage of funds with a β_i greater than X less the percentage of funds with a β_i less than $-X$, where X is a cutoff chosen by the user:

$$C^*_{F_i,t} = a^*_{F_i,t} - b^*_{F_i,t} \qquad (A.90)$$

where $a^*_{F_i,t}$ is the percentage of funds with β_i greater than X over the period $t - 25$ weeks through t and $b^*_{F_i,t}$ is the percentage of funds with β_i less than $-X$ over the period $t - 25$ weeks through t.

F.4.2 Inputs and Outputs

- **Input:** Weekly excess returns for the 107 hedge funds in question from April 2005 to June 2010.

- **Input:** Weekly returns for the four factors described in Section F.4.1 from April 2005 to June 2010. Specifically, the **carry factor** is the Deutsche Bank Currency Harvest G10 Index (Bloomber code: DBHVG10U), the **trend factor** is the AFX Currency Management Index, the **value factor** is the Deutsche Bank G10 Valuation Index (Bloomberg code DBPPPUSF), and the **Currency Volatility Factor** is the first difference of the weekly Deutsche Bank Currency Volatility Index (Bloomberg code is CVIX).

- **Output:** Two measures of crowdedness per factor as defined in equations (A.89) and (A.90).

F.4.3 Empirical Findings

It is interesting to focus on the authors' results in 2008. In the first quarter of 2008, a higher-than-usual percentage of funds were significantly exposed to the carry factor and those funds suffered during the market turbulence in the last quarter of 2008, when carry collapsed.

Similarly, in the first quarter of 2008, a high percentage of funds bet significantly against value. Later in 2008, however, value delivered strong performance, resulting in substantial losses for the contrarians, who were caught wrong-footed.

The story for trend is different: Trend was a crowded strategy in 2005, but this crowdedness simply led to flat performance for the trend strategy during this period. After managers gave up on the trend strategy, it delivered strong performance.

Although their sample period was too short for more formal statistical tests, their analysis suggests that there may be an adverse relationship between crowdedness and factor performance, in particular in the carry and value styles. This phenomenon has also been documented in the equity market neutral hedge fund space (see Khandani and Lo (2007, 2011)).

F.5   Equity Market Illiquidity

Khandani and Lo (2011) propose two distinct measures of equity market liquidity. The authors analyze a contrarian trading strategy which consists of buying losers and selling winners. Such a strategy corrects temporary supply-demand imbalances by providing liquidity. Although it is a profitable strategy, its profitability has been steadily decreasing since the late 1990s, presumably due to the fact that an increasing number of market players have been engaging in this liquidity-providing trade thus reducing the liquidity premium of this trade. A measure of equity market liquidity can be obtained by observing the performance of such a trading strategy: presumably, when it does very well, there is less liquidity in the market and vice versa.

The authors' second measure of market liquidity is related to Kyle's (1985) "lambda", in which liquidity is measured by a linear regression estimate of the volume required to move the price of a security by one dollar.

F.5.1   Definition

1. **Contrarian Strategy Liquidity Measure:** The authors present a simple mean-reversion strategy originally proposed by Lo and MacKinlay (1990a) to proxy for marketmaking (i.e. liquidity provisioning) profits. This high-frequency mean-reversion strategy is based on buying losers and selling winners over lagged m-minute returns, where m is varied from 5 to 60 minutes. Specifically, a long and a short portfolio are formed at time t by looking at the returns of all stocks in the sample over the previous m minutes. The stocks are sorted into performance deciles, and the strategy is to form a portfolio that is long those stocks in the lowest return-decile over this previous m-minute interval, and short those stocks in the highest return-decile over the previous m-minute interval. This dollar neutral portfolio is then held for q minutes, i.e., until time $t + q$, after which the portfolio formation process is repeated by forming deciles over the returns over the previous m minutes, i.e., the interval from time $t + q - m$ to

$t + q$, and then holding these new positions for q minutes, i.e. until time $t + 2q$, and so on.

It should be noted that stocks in both the long and short portfolios are equally weighted and thus the overall portfolio is dollar neutral. Furthermore, no overnight positions are allowed. Finally, the last traded price of a security is used in each m-minute interval to calculate the returns; hence, the first set of prices for each day are the prices based on trades just before 9:30am+m minutes and the first set of positions are established at 9:30am+$2m$ minutes.

2. **Price Impact Liquidity Measure:** This approach is motivated by Kyle's (1985) model in which liquidity is measured by a linear-regression estimate of the volume required to move the price of a security by one dollar. Sometimes referred to as "Kyle's lambda", this measure is an inverse proxy of liquidity, with higher values of lambda implying lower liquidity and market depth. The authors estimate this measure on a daily basis by using all transactions during normal trading hours on each day. Given the sequence of intraday returns $[R_{i,1}, R_{i,2}, \ldots, R_{i,T}]$, prices $[p_{i,1}, p_{i,2}, \ldots, p_{i,T}]$, and volumes $[v_{i,1}, v_{i,2}, \ldots, v_{i,T}]$ for security i during a specific day, the following regression is estimated:

$$R_{i,t} = \hat{c}_i + \hat{\lambda}_i \cdot \text{Sgn}(t) \log(v_{i,t} p_{i,t}) + \epsilon_{i,t} \tag{A.91}$$

where $\text{Sgn}(t) \equiv -1$ or $+1$ depending on the direction of the trade, i.e., "buy" or "sell", as determined according to the following rule: if $R_{i,t}$ is positive, the value $+1$ is assigned to that transaction (to indicate net buying), and if $R_{i,t}$ is negative, the value -1 is assigned to that transaction (to indicate net selling). Any interval with zero return receives the same sign as that of the most recent transaction with a non-zero return (using returns from the prior day, if necessary). The aggregate measure of market liquidity (MLI) is then given by the daily cross-sectional average of the estimated price impact coefficients:

$$\text{MLI} = \frac{\sum_{i=1}^{N} \lambda_i}{N} \tag{A.92}$$

where N is the number of stocks for which the measure is calculated on that day.

F.5.2   Inputs and Outputs

- **Input:** For both measures, the NYSE Trade and Quote (TAQ) data from July 2, 2007 to September 28, 2007, are used for all stocks in the S&P 1500. Only the data from actual trades are used as reported in the "Daily Trades File" of TAQ. Furthermore, only trades that occur during normal trading hours (9:30am to 4:00pm) are used. The raw data used from TAQ are transaction prices and volumes.

For the "contrarian strategy liquidity measure", the time t portfolio formation is based on the returns of the previous m minutes. These returns are calculated by using the last transaction price in the $[t - m, t]$ interval and comparing it to the last transaction

price in the previous interval $[t - 2m, t - m]$. After the portfolio is formed, it is held for q minutes. The authors vary both m and q from 5 to 60 minutes.

For the "price impact liquidity measure", the returns are calculated on a tick-by-tick basis.

- **Output:** The "contrarian strategy liquidity measure" described in Part 1 of Section F.5.1 and the "price impact liquidity measure" in equations (A.91) and (A.92).

F.5.3 Empirical Findings

The authors find that increases in the price impact liquidity measure were common to all market-cap groups and not limited to the smaller stocks. Furthermore, a substantial drop in liquidity in the days leading up to August 6, 2007 was documented. Moreover, this measure of liquidity was at its lowest level in the week following the week of August 6. This pattern suggests that some marketmakers, burned by the turn of events in the week of August 6, reduced their marketmaking capital in the following days and in turn caused the price impacts to substantially rise starting on August 10 and remain high for the following week. As quantitative factor portfolios were being deleveraged and unwound during the last two weeks of July 2007, the liquidity premium for the contrarian trading strategy increased, implying higher profits for marketmaking strategies. Indeed, the authors find that during the two-week period after the week of August 6, the returns of the contrarian trading strategy are substantially higher than their norm during the whole sample period.

A comprehensive discussion of different theoretical and empirical aspects of liquidity with an overview of the most relevant studies in the area can be found in Amihud (2002) and Hasbrouck (2007).

F.6 Serial Correlation and Illiquidity in Hedge Fund Returns

It is a well documented empirical fact that returns to hedge funds and other alternative investments are often highly serially correlated, and Getmansky, Lo, and Makarov (2004) explore several sources of such serial correlation and show that the most likely explanation is illiquidity exposure and smoothed returns. The authors propose an econometric model of return smoothing coefficients; they find that these vary considerably across hedge-fund style categories and posit that they may be a useful proxy for quantifying illiquidity exposure. Their argument for why this is the case goes as follows: for illiquid securities, prices are not readily available and often, linear extrapolation methods of pricing are used, resulting in more persistent and smoother returns. Even when quotes are obtained from broker/dealers, the phenomenon still exists because broker/dealers also often use linear extrapolation pricing methods; furthermore, averaging quotes from multiple broker/dealers can lead to smoother profiles. Thus, hedge funds which invest in more illiquid securities/areas should exhibit a higher serial correlation in their reported returns.

F.6.1 Definition

The basic model for hedge fund returns presented in the paper differentiates between true returns R_t and observed returns R_t^o, i.e., the returns that hedge funds report to their investors.

The observed return is a weighted average of the fund's true returns over time:

$$
\begin{aligned}
R_t^o &= \theta_0 R_t + \theta_1 R_{t-1} + \cdots + \theta_k R_{t-k} \quad, \quad \theta_j \in [0,1] \quad, \quad j = 0, \ldots, k \\
1 &= \theta_0 + \theta_1 + \cdots + \theta_k \ .
\end{aligned}
\tag{A.93}
$$

We call the above model the "smoothing" model. Under the above specification, one can immediately see that:

$$
\begin{aligned}
\mathrm{E}[R_t^o] &= \mathrm{E}[R_t] \\
\mathrm{Var}[R_t^o] &\leq \mathrm{Var}[R_t] \\
\mathrm{Corr}[R_t^o, R_{t-m}^o] &= \frac{\mathrm{Cov}[R_t^o, R_{t-m}^o]}{\mathrm{Var}[R_t^o]} = \frac{\sum_{j=0}^{k-m} \theta_j \theta_{j+m}}{\sum_{j=0}^{k} \theta_j^2} \ .
\end{aligned}
\tag{A.94}
$$

Thus, although the observed mean returns are the same as the true mean returns, smoothing results in a lower observed variance and a higher serial correlation. Funds with θs that are more "spread out" thus engage in more returns smoothing. The measure which quantifies this effect is:

$$
\xi \equiv \sum_{j=0}^{k} \theta_j^2 \in [0,1] \ .
\tag{A.95}
$$

The smaller ξ is, the more smoothing is done. Thus, the question becomes how to estimate the θs. The authors propose two methods of doing this: the maximum likelihood approach (MLE) and the regression approach.

1. **MLE approach:** Given the specification of the smoothing process in (A.93), the θs can be estimated using maximum likelihood in a fashion similar to the estimation of standard moving-average (MA) time-series models (see Brockwell and Davis (1991, Chapter 8)). Let X_t denote the de-meaned observed return series. The model in (A.93) implies the following properties for X_t:

$$
\begin{aligned}
X_t &= \theta_0 \eta_t + \theta_1 \eta_{t-1} + \cdots + \theta_k \Lambda_{t-k} \\
1 &= \theta_0 + \theta_1 + \cdots + \theta_k \\
\eta_k &\sim N(0, \sigma_\eta^2) \ .
\end{aligned}
\tag{A.96}
$$

Denote the given set of realized observations as $\mathbf{X} = [X_1 \cdots X_T]'$. Let $\hat{\mathbf{X}} = [\hat{X}_1 \cdots \hat{X}_T]'$ where $\hat{X}_1 = 0$ and $\hat{X}_j = E[X_j | X_{j-1}, \ldots, X_1]$ where $j \geq 2$, and define the mean squared

error r_t of the predicted value of X_{t+1} as:

$$r_t = \frac{E[(X_{t+1} - \hat{X}_{t+1})^2]}{\sigma_\eta^2} . \tag{A.97}$$

Given the above notation, it can be shown that the estimator of the variance is:

$$\hat{\sigma}_\eta^2 \equiv S(\theta) = \frac{\sum_{t=1}^T (X_t - \hat{X}_t)^2}{T r_{t-1}} . \tag{A.98}$$

Armed with the above expression, the likelihood function which needs to be maximized over the parameter space of θs is:

$$L(\theta) = \log(S(\theta)) + T^{-1} \sum_{t=1}^T \log(r_{t-1}) . \tag{A.99}$$

The function $L(\theta)$ is maximized subject to the following constraints:

- **Normalization Constraint:** $\theta_0 + \theta_1 + \cdots + \theta_k = 1$.
- **Invertibility Constraint:** Since the process for X_T should be an invertible moving-average process, the estimated θs need to satisfy $\theta_1 < (1/2)$, $\theta_2 < (1 - \theta_1)/2$, etc.

The authors mention that they use the Matlab optimization toolbox in solving the MLE problem. They also mention that they verify the results by running the MA(k) estimation routine in STATA.

2. **The regression approach:** If one is willing to impose a linear factor model on the true returns process of the following form:

$$R_t = \mu + \beta \Lambda_t + \epsilon_t, \qquad E[\Lambda_t] = E[\epsilon_t] = 0, \qquad \epsilon_t, \Lambda_t \sim IID \tag{A.100}$$

where the true return depends on a common factor Λ_t, has a mean of μ, and a variance of σ^2, then there is a much simpler method of estimating the θs. By substituting (A.100) into (A.93), the observed returns can be written as a function of lagged factors:

$$
\begin{aligned}
R_t^0 &= \mu + \beta(\theta_0 \Lambda_{t-1} + \cdots + \theta_k \Lambda_{t-k}) + u_t \\
&= \mu + \gamma_0 \Lambda_{t-1} + \cdots + \gamma_k \Lambda_{t-k} + u_t \\
u_t &= \theta_0 \epsilon_t + \theta_1 \epsilon_{t-1} + \cdots + \theta_k \epsilon_{t-k} .
\end{aligned}
\tag{A.101}
$$

By running the above regression, one can estimate the γs and thus the θs:

$$\hat{\beta} \; = \; \hat{\gamma}_0 \, + \, \hat{\gamma}_1 \, + \, \cdots \, + \, \hat{\gamma}_k, \quad \hat{\theta}_j \; = \; \hat{\gamma}_j/\hat{\beta} \, . \qquad\qquad (A.102)$$

While the second method for estimating θs is much easier, its downside is that it assumes a specific linear factor structure; specifying the correct factor to use for a given hedge fund entails factor mis-specification risks. Furthermore, even under a reasonable factor, because it is an MA process, the ordinary least squares (OLS) estimators will not be efficient and the usual standard errors are incorrect.

The MLE approach is more general and can yield reliable standard errors at the cost of being more complex and assuming normality of the ηs.

F.6.2 Inputs and Outputs

The authors apply their estimation methods to hedge fund return data using $k = 2$ lags.

- **Input:** Monthly hedge fund return data from the TASS database from November 1977 to January 2001. They keep only hedge funds that have at least 5 years of data history. The authors also run their algorithm on the CSFB/Tremont hedge fund category indexes over the same period.

- **Input:** For the regression case, the appropriate factor Λ_t for a hedge fund must be selected. The authors use the monthly returns for the S&P 500, although they realize that this may not be appropriate for some funds.

- **Output:** The first-, second-, and third-order autocorrelations for each hedge fund's returns.

- **Output:** Each fund's θs and ξ as defined in (A.95).

F.6.3 Empirical Findings

The authors find that hedge funds that, by common wisdom and self-description, invest in more illiquid securities, such as Convertible Arbitrage funds, Fixed Income Directional funds, and Emerging Market funds, have higher serial autocorrelations, lower values of ξ, and lower values of θ_0 compared to funds which typically invest in very liquid securities, such as U.S. Equity funds and Managed Futures funds, whose autocorrelations are near zero and have values of θ_0 and ξ close to 1.

Furthermore, the authors examine whether phenomena other than the returns smoothing model could be causing such high autocorrelations in illiquid funds, such as time varying expected returns, time varying leverage, and incentive fee structure; they show that none of these can produce the empirically observed high levels of returns persistence.

Finally, although the authors focus on hedge funds, their analysis may be applied to other investments and asset classes, e.g., real estate, venture capital, private equity, art and other collectibles, and other assets for which illiquidity and smoothed returns are even more problematic. More generally, their econometric model may be applied to a number of other contexts in which there is a gap between reported results and economic realities.

Further reading on the moving average model presented here can be found in Lo and MacKinlay (1988, 1990a).

F.7 Broader Hedge-Fund-Based Systemic Risk Measures

Chan, Getmansky, Haas, and Lo (2006a, 2006b) consider the broader impact of hedge funds on systemic risk by examining the unique risk/return profiles of hedge funds at both the individual-fund and aggregate-industry levels and propose three new risk measures for hedge fund investments. The first measure is an autocorrelation-based measure used to proxy hedge fund illiquidity exposures similar to that in Getmansky, Lo, and Makarov (2004) (see Section F.6 for a summary). The second measure quantifies the probability of liquidation of a hedge fund, and the third measure is a regime-switching-based model used to quantify the level of aggregate distress in the hedge fund sector. The nice feature of the above measures is that they are indirect measures of risk which require hedge fund returns and sizes as the primary inputs, making them easy to implement from a data-requirements perspective.

F.7.1 Definition

The three metrics presented in the paper are described below:

1. **Autocorrelation-Based Measures:** For a given monthly return series of a hedge fund, the first 6 autocorrelation coefficients are estimated. The Q statistic, proposed by Ljung & Box in Ljung and Box (1978) is then calculated according to the formula:

$$Q \;=\; T(T+2)\sum_{j=1}^{k} \hat{\rho}_j^2/(T-j) \; . \tag{A.103}$$

   The Q statistic is asymptotically distributed as a χ_k^2 under the null hypothesis of no autocorrelation. Funds with large positive autocorrelations will exhibit large Q statistics.

   In terms of defining an overall measure of systemic risk (or illiquidity) in the hedge fund sector, the authors propose using a cross-sectional weighted average of hedge funds' rolling first-order autocorrelations. Let $\rho_{t,i}$ denote hedge fund i's first-order autocorrelation in month t using a window of past returns (the authors use 36 months). The aggregate measure of illiquidity ρ_t^* is given by:

$$\rho_t^* \;\equiv\; \sum_{i=1}^{N_t} \omega_{it}\rho_{t,i} \tag{A.104}$$

   where N_t denotes the number of hedge funds in the sample at time t and ω_{it} denotes

the weight of hedge fund i, given by:

$$\omega_{it} \equiv \frac{AUM_{it}}{\sum_{j=1}^{N_t} AUM_{jt}} \tag{A.105}$$

where AUM_{jt} are the assets under management for fund j at time t.

2. **Hedge Fund Liquidation Probability:** The authors create a measure of the probability of hedge fund liquidation by running a logit model on a set of factors driving hedge fund performance. The TASS database of individual hedge fund returns contains two subsets: the live database and the graveyard database. Roughly speaking, the graveyard database contains hedge funds which have stopped reporting data to TASS. Thus, a hedge fund that starts out in the live database may end up in the graveyard database. Hedge funds in the graveyard database are considered to be liquidated although this may not necessarily be the case; for a thorough description of the TASS database categorizations, see Chan, Getmansky, Haas, and Lo (2006a) or Getmansky, Lo, and Makarov (2004). For tractability, the authors focus on annual observations only, so the dependent variable Z_{it} indicates whether fund i is alive ($Z_{it} = 0$) or liquidated ($Z_{it} = 1$) in year t. Thus, for each hedge fund, its time series for Z_{it} will look like $[0, 0, \ldots, 0, 1]$. Associated with each Z_{it} is a set of explanatory variables listed below in Table A.8.

Variable	Definition
AGE	Current age of fund in months
ASSETS	Log of fund's AUM
ASSETS_{-1}	Log of fund's AUM as of December 31 of previous year
RETURN	YTD return for fund
RETURN_{-1}	Total returns last year
RETURN_{-2}	Total return two years ago
FLOW	Fund's current YTD total dollar inflow
	divided by previous year's AUM, where
	inflow in month k is: $\text{FLOW}_k = \text{AUM}_k - \text{AUM}_{k-1}(1 + R_k)$
	where R_k is month k's return
FLOW_{-1}	Last year's inflow over AUM the year before
FLOW_{-2}	Inflow 2 years ago over AUM 3 years ago

Table A.8: Explanatory Variables Used In Estimating Liquidation Probabilities

Therefore, the logit model to be estimated is:

$$\begin{aligned}
Z_{it} = G\Big(& \beta_0 + \beta_1 \text{AGE}_{it} + \beta_2 \text{ASSETS}_{it-1} + \\
& \beta_3 \text{RETURN}_{it} + \beta_4 \text{RETURN}_{it-1} + \beta_5 \text{RETURN}_{it-2} + \\
& \beta_6 \text{FLOW}_{it} + \beta_7 \text{FLOW}_{it-1} + \beta_8 \text{FLOW}_{it-2} + \epsilon_{it} \Big).
\end{aligned} \tag{A.106}$$

134

The above model is estimated via the standard maximum likelihood method for logit models (see, for example, Woolridge (2002)). After the betas have been estimated, one can estimate the probability of liquidation for a hedge fund at time t as:

$$\hat{p}_{it} = \frac{\exp(X_{it}'\hat{\beta})}{1 + \exp(X_{it}'\hat{\beta})} . \tag{A.107}$$

The indicator of systemic risk across the hedge fund industry is the mean and/or median of each fund's $\hat{p}_{it}$.

3. **Regime-Switching-Based Systemic Risk Measure:** The authors use a two state regime-switching model. Denote by R_t the return of a hedge fund index in period t and suppose it satisfies the following:

$$R_t = I_{it} \cdot R_{1t} + (1 - I_t) \cdot R_{2t} , \quad R_{it} \sim \mathcal{N}(u_i, \sigma_i^2) \tag{A.108}$$

$$I_t = \begin{cases} 1 & \text{with probability } p_{11} \text{ if } I_{t-1} = 1 \\ 1 & \text{with probability } p_{21} \text{ if } I_{t-1} = 0 \\ 0 & \text{with probability } p_{12} \text{ if } I_{t-1} = 1 \\ 0 & \text{with probability } p_{22} \text{ if } I_{t-1} = 0 \end{cases} \tag{A.109}$$

The transition probabilities p_{ij}, along with the means and variances of R_{1t} and R_{2t}, are estimated via maximum likelihood of the above model. Note that the specification of the process I_t is a Markov Chain. Because the k-step transition matrix of a Markov chain is simply given by $\mathbf{P^k}$, the conditional probability of the regime I_{t+k} given the returns up to time t is:

$$\text{Prob}(I_{t+k} = 1|R_t) = \pi_1 + (p_{11} - p_{21})^k [\text{Prob}(I_t = 1|R_t) - \pi_1] \tag{A.110}$$

$$\pi_1 \equiv \frac{p_{21}}{p_{12} + p_{21}} \tag{A.111}$$

where $\text{Prob}(I_t = 1|R_t)$ is the probability that the date t regime is 1 given the historical data up to and including date t. Using similar recursions of the Markov chain, the conditional expectation of R_{t+k} is:

$$\begin{aligned} \text{E}[R_{t+k}|R_t] &= a_t'\mathbf{P_k}\mu \\ a_t &= [\text{Prob}(I_t = 1|R_t) \, \text{Prob}(I_t = 2|R_t)]' \\ \mu &\equiv [\mu_1 \, \mu_2]' . \end{aligned} \tag{A.112}$$

Thus, once the above model is estimated, the hedge fund systemic risk indicator (HF-SRI) for the overall hedge fund industry is calculated as the sum of the probabilities

of being in the low-mean state at time t for each hedge fund index:

$$\text{HFSRI}_t \quad = \quad \sum_{i=1}^{n} \text{Prob}(I_t^i = \text{low-mean state of } i | R_t^i) \qquad (\text{A.113})$$

where n denotes the number of hedge fund indexes. Note that the summed probabilities, even if renormalized to lie in the unit interval, cannot be interpreted formally as a probability because the regime-switching process was specified individually for each index, not jointly across all indexes. Therefore, the interpretation of the low-mean state for convertible arbitrage may be quite different than the interpretation of the low-mean state for equity market neutral. Nevertheless, as an aggregate measure of the state of the hedge fund industry, the summed probabilities may contain useful information about systemic risk exposures.

F.7.2 Inputs and Outputs

Below, we categorize the inputs and outputs needed for each of the 3 systemic risk measures presented.

1. **Autocorrelation-Based Measures:**

   - **Input:** Monthly hedge fund return data and AUM for all funds in the TASS database with at least 36 consecutive months of non-missing returns, along with the number of funds each month (at the bottom, measured by the right vertical axis), from January 1977 to August 2004.

   - **Ouput:** For each hedge fund, its Q statistic defined in (A.103).

   - **Output:** The systemic liquidity indicator ρ_t^* defined in (A.104).

2. **Hedge Fund Liquidation Probability:**

   - **Input:** Using all funds in both the live and graveyard databases of the TASS database for which there are at least 2 years of history, collect the data in Table A.8 from January 1994 to August 2004.

   - **Ouput:** The probability of liquidation for each hedge fund $\hat{p}_{it}$ as defined in equation (A.107).

3. **Regime-Switching-Based Systemic Risk Measure:**

   - **Input:** The monthly returns of each of the 14 CSFB/Tremont hedge fund indexes from January 1994 to August 2004.

   - **Output:** The hedge fund systemic risk indicator HSFRI at time t as defined in (A.113).

   - **Output:** Predictions about the expected return of the index as well as the probability of being in each state at time $(t + k)$ (equations (A.112) and (A.110) respectively).

F.7.3 Empirical Findings

The authors find that the hedge fund liquidity indicator ρ_t^* shows considerable swings over time, with dynamics that seem to be related to liquidity events. Furthermore, over the latter period of their sample (2001–2004), this indicator is on the rise, implying that hedge funds are taking on more illiquidity exposure.

In terms of the hedge fund liquidation probabilities, they find that the average liquidation probability for funds in 2004 (end of their sample) is over 11%, higher than the historical unconditional attrition rate of 8.8%. A higher attrition rate is not surprising for a rapidly growing industry, but it may foreshadow potential instabilities that can be triggered by seemingly innocuous market events.

When running the regime-switching model on the various hedge fund categories, it is interesting to note that the two state pairings are not always (high mean, low volatility) and (low mean, high volatility). In fact, for equity market neutral, global macro, and long/short equity, the pairings are the opposite, indicating that these strategies perform well in high volatility states. HFSRI has local maxima in 1994 and 1998 as expected, but there is a stronger peak around 2002, largely due to equity market neutral, global macro, and long/short equity funds. This pattern corresponds remarkably well to the common wisdom that, over that period, these three strategies underperformed. In 2004, the probability of being in the low-mean state is high for a number of hedge fund indexes, foreshadowing increased leverage and ultimately increased systemic risk.

G Matlab Program Headers

In this section, we provide a listing of the headers for all the Matlab programs available to compute the risk analytics described in this survey. The complete sourcecode for these programs is available at `http://`.

G.1 Function `delta_co_var`

```
function dcovar = delta_co_var(output_returns, input_returns,
    lagged_factors_returns, quantile)
%
% Based on the paper "CoVar", Tobias Adrian and Markus K. Brunnermeier.
% Calculates the delta_covar of an output institution (or the system) on
% another institution.
%
% PARAMETERS:
% output_returns - The returns of the output institution or system.
%   An (nx1)-vector.
% input_returns - The returns of the input institution whose contribution
%   we want to quantify. An (nx1)-vector.
% lagged_factors_returns - The lagged returns (lag = 1) for the factors as
%   used in eq. 6 of the paper. An ((n+1)xk)-matrix.
% quantile - The quantile we want to use (0.05 or 0.01) in the paper.
```

G.2 Function `quantile_regression`

```
function betas = quantile_regression(y,X,q)
%
% Calculates the q-quantile regression coefficients.
%
% PARAMETERS:
% y - The response variable. An (nx1)-vector.
% X - the matrix of regressors An (nxk)-vector.
% q - the quantile i.e. 0.5 for median regression.
```

G.3 Function `co_risk`

```
function c = co_risk(output_cds_spreads,input_cds_spreads,
    risk_factors_series,q,risk_factors_values )
%
% Calculates the conditional co-risk between two institutions.
%
% PARAMETERS:
% output_cds_spreads - The cds-spreads time series for the output
%   institution. An (nx1)-vector.
% input_cds_spreads - The cds-spreads time series for the input institution.
%   An (nx1)-vector.
% risk_factors_series - Risk-factors time series. An (nxk)-matrix.
% q - The quantile. The paper uses q = 0.95.
% risk_factors_values - the values of the risk factors at the period we
%   calculate the co-risk. A (1xk)-vector.
```

G.4 Function `turbulence`

```
function [turbulence_series, threshold, turbulent_periods] =
    turbulence(asset_returns,q)
%
% Calculates the turbulence of each period, the q-percentile of turbulence
% distribution and the turbulent periods.
%
% PARAMETERS:
% asset_returns - A matrix of asset returns. Rows are different dates.
% Columns are different assets.
% q - The percentile. In the paper q = 0.75.
%
% OUTPUTS:
% turbulence_series - The turbulence for each period in the sample.
% threshold - The q-percentile of turbulence distribution.
% turbulent_periods - The periods where turbulence > quantile.
```

G.5   Function `turbulence_var`

```
function var = turbulence_var(asset_returns,portfolio, q)
%
% Calculates the value-at-risk of a portfolio using only data from
% turbulent periods. Turbulence is defined at a q-percentile.
%
% PARAMETERS:
% asset_returns - A matrix of asset returns. Rows are different dates.
%    Columns are different assets.
% portfolio - The portfolio weights.
% q - The percentile.
```

G.6   Function `marginal_expected_shortfall`

```
function mes = marginal_expected_shortfall(firm_returns, market_returns)
%
% Calculates the marginal expected shortfall of a firm.
%
% PARAMETERS:
% firm_returns - The time series of returns for the firm.
% market_returns - The time series of returns for the market.
```

G.7   Function `leverage`

```
function lvg = leverage(book_assets, book_equity, market_equity)
%
% Calculates the standard approximation of leverage for a firm.
%
% PARAMETERS:
% book_assets - The book assets of the firm.
% book_equity - The book equity of the firm.
% market_equity - The market equity of the firm.
```

G.8   Function `systemic_expected_shortfall`

```
function ses = systemic_expected_shortfall(mes_training_sample,
    lvg_training_sample, ses_training_sample, mes_firm, lvg_firm)
%
% Calculates the systemic expected shortfall for a firm.
%
% PARAMETERS:
% mes_training_sample - The marginal expected shortfalls for the training
%    sample of firms. An (nx1)-vector.
% lvg_training_sample - The leverages for the training sample of firms.
%    An (nx1)-vector.
```

```
% ses_training_sample - The systemic expected shortfalls for the training
%    sample of firms. An (nx1)-vector.
% mes_firm - The marginal expected shortfall for the firm.
% lvg_firm - The leverage for the firm.
```

G.9   Function `contrarian_trading_strategy`

```
function ret = contrarian_trading_strategy(input_returns, realized_returns)
%
% Calculates the cumulative return of the contrarian trading strategy.
%
% PARAMETERS:
% input_returns - The returns of the securities used to calculate the weights
%    of the strategy. Rows are the different periods. Columns are the
%    different securities.
% realized_returns - The realized returns of the securities to calculate
%    the performance of the strategy. Rows are the different periods.
%    Columns are the different securities.
```

G.10   Function `kyles_lambda`

```
function mli = kyles_lambda(returns, prices, volumes)
%
% Calculates Kyle's lambda (price impact liquidity measure).
%
% PARAMETERS:
% returns - The returns of different securities. Rows are different dates
%    and columns are different securities.
% prices - The closing prices of the securities. Rows are different dates
%    and columns are different securities.
% voumes - The trading volumes of the securities. Rows are different dates
%    and columns are different securities.
```

G.11   Function `systemic_liquidity_indicator`

```
function [q_stats sys_liquidity_ind] =
    systemic_liquidity_indicator(funds_returns,assets_under_management)
%
% Calculates the Q-stats for each fund and the systemic liquidity indicator.
%
% PARAMETERS:
% funds_returns - The monthly returns of the funds. A (nxk)-matrix.  Rows
%    are the different periods.  Columns are the different funds.
% assets_under_management - The assets under management for the different
%    funds. A (kx1)-vector.
%
```

```
% OUTPUTS:
% q_stats - The q-statistics for the different funds' returns.
%    A (kx1)-vector.
% sys_liquidity_ind - The systemic liquidity indicator.
```

G.12 Function `probability_liquidation_model`

```
function coeffs = probability_liquidation_model(aums, returns, flows,
    is_liquidated_series)
%
% Calculates the coefficients in the logit model for probability of
% liquidation of a fund.
%
% PARAMETERS:
% aums - The series of AUMS for the different funds. An (nxk)-matrix. Rows
%    are the different months. Columns are the different funds.
% returns - The series of returns for the different funds. An (nxk)-matrix.
%    Rows are the different months. Columns are the different funds.
% flows - The series of flows for the different funds. A (nxk)-matrix. Rows
%    are the different months. Columns are the different funds.
% is_liquidated_series - A binary (nxk)-matrix . If the (i,j) component = 1
%    it denotes that fund j is liquidated at period i.
```

G.13 Function `crowded_trades`

```
function crowdedness  = crowded_trades(funds_returns, factor_returns,
factor_index, threshold)
%
% Calculates the crowdedness series of a factor.
%
% PARAMETERS:
% funds_returns - A matrix of funds returns. Rows are different dates.
%    Columns are different assets.
% factor returns - A matrix of funds returns. Rows are different dates.
%    Columns are different assets. For the paper there are 4 factors,
%    i.e., 4 cols.
% factor_index - The factor whose crowdedness we want to calculate
%    threshold. If it is specified then crowdedness is found with
%    respect to beta greater than threshold.
```

G.14 Function `cca`

```
function [put_price systemic_risk_indicator_contribution] = cca(equity,
    volatility,risk_free_rate,default_barrier,time_to_maturity,cds_spread)
%
% Based on the paper "Systemic CCA - A Model Approach to Systemic Risk" by
```

```
% D. F. Gray, A.A. Jobst.
% Calculates the price of the put option and the contribution  of the
% company to the systemic risk indicator suggested in the paper.
%
% PARAMETERS:
% equity - The market value of the equity of the company.
% volatility - The volatility of the equity.
% risk_free_rate - The risk free rate.
% default_barrier - Face value of the outstanding debt at maturity.
% time_to_maturity - Time to maturity of the debt.
% cds_spread - The cds spread.
%
% OUTPUTS:
% put_price - The price of the put
% systemic_risk_indicator_contribution - The contribution of this firm to
%   the systemic risk indicator
```

G.15   Function `distressed_insurance_premium`

```
function dip = distressed_insurance_premium(default_probabilities,
    correlations)
%
% Calculates the distressed insurance premium.
%
% PARAMETERS:
% default_probabilities - The default probabilities of the banks.
%   An (nx1)-vector.
% correlations - The correlation matrix of the assets' returns of the banks.
```

G.16   Function `credit_funding_shock`

```
function [capital_losses defaulted_banks] =
credit_funding_shock(default_bank,
capitals, interbank_loans, lambda, rho, delta)
%
% Simulates a credit and funding shock in the banking system.
%
% PARAMETERS:
% default_bank - The bank that is initially triggered to default. An integer
%   1<=default_bank <=n.
% capitals - The capitals of the banks. An (nx1)-vector. A bank defaults if
%   its capital becomes negative.
% interbank_loans - An (nxn)-matrix describing the loans between banks. The
%   (i,j)-component denotes a loan from bank j to bank i.
% lambda - The loss given default parameter. In the paper it is set to 1.
% rho - The loss of funding fraction. In the paper it is set to 0.35.
```

```
% delta - The loss parameter due to forced selling. In the paper it is set
%   to 1.
%
% OUTPUTS:
% capital_losses - The capital losses of the banks.
% defaulted_banks - The banks that default. An (nx1)-binary vector where 1
%   denotes default.
```

G.17   Function `pca`

```
function [Sigma, eigenvalues, eigenvectors] = pca(asset_returns)
%
% Calculates the covariance matrix of
% the returns for different assets and its eigenvalues and eigenvectors.
%
% PARAMETERS:
% asset_returns - The time series of asset returns. An (nxk)-matrix. Rows
%   are the different dates. Columns are the different assets.
%
% OUTPUTS:
% Sigma - The covariance matrix of the asset returns.
% eigenvalues - The eigenvalues of the covariance matrix.
% eigenvectors - A matrix with the corresponding eigenvectors of the
%   covariance matrix as its columns.
```

G.18   Function `linear_granger_causality`

```
function  p_value = linear_granger_causality(input_institution_returns,
    output_institution_returns)
%
% Calculates the p-value of the linear Granger-causal relationship between
% input_institution_returns and output_institution_returns.
%
% PARAMETERS:
% input_institution_returns - The time series returns of the input
%   institution. An (nx1)-vector.
% output_institution_returns - The time series returns of the output
%   institution. An (nx1)-vector.
```

G.19   Function `hac_regression`

```
function [betas, V_hat] = hac_regression(y,X,
    truncation_lag_to_observations_ratio)
%
% Calculates the regression coefficients and the HAC ("heteroskedasticity
% and autocorrelation consistent") estimator.
```

```
%
% PARAMETERS:
% y - The dependent variable An (nx1)-vector.
% X - The covariates. An (nxk)-vector.
% truncation_lag_to_observations_ratio - The truncation lag to number of
%   observations ratio used to construct the HAC estimator.
%
% OUTPUTS:
% betas - The regression coefficients. A (kx1)-vector.
% V_hat - The HAC estimator.
```

G.20 Function dynamic_causality_index

```
function [connection_matrix_robust, connection_matrix, dci] =
    dynamic_causality_index(asset_returns,statistical_significance_threshold)
%
% Based on the paper "Econometric Measures of Systemic Risk in the Finance
% and Insurance Sectors" by M. Billio, M. Getmansky, A.W. Lo, L. Pelizzon
% Calculates the dynamic causality index and the adjacency matrix of linear
% Granger causal relationships for different institutions based on their
% returns.
%
% PARAMETERS:
% asset_returns - The time series of institutions returns. An (nxk)-matrix
%   Rows are the different dates. Columns are the different institutions.
% statistical_significance_threshold - The threshold for p-value that
%   determines if the linear Granger-causal relationship is statistically
%   significant. Usually 0.05 or 0.01.
%
% OUTPUTS:
% connection_matrix_robust - The adjacency matrix describing the linear
%   Granger-causal relationships among the institutions. If
%   connection_matrix(i,j) = 1 then institution i affects institution j.
%   It corrects for autocorrelations and heteroskedasticity.
% connection_matrix - The adjacency matrix describing the linear Granger-
%   causal relationships among the institutions. If
%   connection_matrix(i,j) = 1 then institution i affects institution j.
%   It does not correct for autocorrelations and heteroskedasticity.
% dci - The dynamic causality index for the robust matrix.
```

G.21 Function dijkstra

```
function distances = dijkstra(A,node)
%
% Implements the Dijkstra algorithm; returns the shortest path lengths
%   between the initial and all other nodes.
```

```
%
% PARAMETERS:
% A - The (nxn) adjacency matrix.
% node - The initial node from which we calculate all the shortest paths to
%   any other node.
```

G.22 Function `calc_closeness`

```
function cl = calc_closeness(adjacency_matrix, node)
%
% Calculates the closeness of a node in a network.
%
% PARAMETERS:
% adjaceny_matrix - The adjacency matrix of the network.
% node - The node.
```

G.23 Function `network_measures`

```
function [in_connections, out_connections, in_out_connections, in_from_other,
    out_to_other, in_out_other, closeness, eigenvector_centrality]=
    network_measures(adjacency_matrix, groups)
%
% For all the nodes of the network it calculates a few network measures
%
% PARAMETERS:
% adjacency_matrix - The adjacency matrix describing the network.
%   An (nxn)-matrix.
% groups - A vector describing the different categories of nodes. i.e. if
%   group = [2 6] that means that we have 3 groups of nodes. Group 1
%   nodes 1,2, Group 2 nodes 3,4,5,6 Group 3 nodes 7 until n.
%
% OUTPUTS:
% in_connections - For each node the number of incoming links.
% out_connections - For each node the number of outcoming links.
% in_out_connections - For each node the sum of incoming and outcoming links.
% in_from_other - For each node the number of incoming links from nodes in
% different categories.
% out_to_other - For each node the number of outcoming links to nodes in
%   different categories.
% in_out_other - For each node the sum of in_from_other and out_to_other.
% closeness - For each node the average shortest path lengths to reachable
%   nodes.
% eigenvector_centrality - For each node its eigenvector centrality.
```

G.24 Function `absorption_ratio`

```
function  ar = absorption_ratio(asset_returns, fraction_eigenvectors)
%
% Calculates the absorption ratio for a time series of asset returns
%
% PARAMETERS:
% asset_returns - A matrix of asset returns. Rows are different dates.
%   Columns are different assets. In the paper number of rows = 500.
% fraction_eigenvectors - The fraction of eigenvectors used to calculate the
%   absorption ratio. In the paper it is 0.2.
```

G.25 Function `dar`

```
function ar_shift = dar(absorption_ratios, fraction)
%
% Calculates the standardized AR shift
%
% PARAMETERS:
% absorption_ratios - A vector of absorption ratios. In the paper
%   absorption ratios are given for 1 year.
% fraction - Number of days in the short term absorption ratio over number
%   of days in the long term absorption ratio. In the paper 15/252 if we
%   consider a year having 252 absorption ratios.
```

G.26 Function `undirected_banking_linkages`

```
function undirected = undirected_banking_linkages(bank_claims,
    non_bank_claims, non_bank_liabilities)
%
% Calculates the undirected network of banking linkages for n
% countries.
%
% PARAMETERS:
% bank_claims - An (nxn)-matrix. Its (i,j) component shows the bank claims
%   from country i to banks in country j.
% non_bank_claims - An (nxn)-matrix. Its (i,j) component shows the claims
%   from banks in country i to non-banks in country j.
% non_bank_liabilities - An (nxn)-matrix. Its (i,j) component shows the
%   liabilities of banks in i to non-banks in j.
```

G.27 Function `directed_banking_linkages`

```
function directed = directed_banking_linkages(initial_net_bank_claims,
    initial_net_non_bank_claims, final_net_bank_claims,
    final_net_non_bank_claims)
```

```
%
% Calculates the diretced network of banking linkages for n
%   countries between two time instants t1 and t2.
%
% PARAMETERS:
% initial_net_bank_claims - An (nxn)-matrix. Its (i,j) component shows the
%   net bank claims from country i to banks in country j at t1. Antisymmetric
%   matrix.
% initial_net_non_bank_claims - An (nxn)-matrix. Its (i,j) component shows
%   the net claims from banks in country i to non-banks in country j at t1.
% final_net_bank_claims - An (nxn)-matrix. Its (i,j) component shows the
%   net bank claims from country i to banks in country j at t2. Antisymmetric
%   matrix.
% final_net_non_bank_claims - An (nxn)-matrix. Its (i,j) component shows the
%   net claims from banks in country i to non-banks in country j at t2.
```

G.28   Function optimal_gap_thresholds

```
function [nts_thresholds, true_positive_thresholds] =
    optimal_gap_thresholds(indicators_series, is_crisis_series,horizon,
    max_thresholds)
%
% Based on the paper "Towards an operational framework for financial
% stability:"fuzzy" measurement and its consequences by C. Borio, M.
% Drehmann.
%
% Calculates the optimal gap thresholds according to nts ratio and
% true positive rate.
%
% PARAMETERS:
% indicators_series - The joint signal indicators yearly time series.
%   An (nx2)-matrix.
% is_crisis_series - A binary nx1 vector. 1 signals that there is a crisis
%   horizon. In the paper it is set 1 to 3 years.
% max_thresholds - A (2x1)-vector with the maximum thresholds for the two
%   indicators. They are integer values.
%
% OUTPUTS
% nts_thresholds - The optimal thresholds for the two indicators when the
%   objective function is nts. A (2x1)-vector.
% true_positive_thresholds - The optimal thresholds for the two indicators
%   when the objective function is true_positive rate. A (2x1)-vector.
```

G.29   Function joint_gap_indicators

```
function [nts num_predicted_crises] =
```

```
    joint_gap_indicators(indicators_series, thresholds, is_crisis_series,
    horizon)
%
% Calculates the signal to noise ratio and the number of predicted crises
% of joint signal indicators for particular thresholds
%
% PARAMETERS:
% indicators_series - The joint signal indicators yearly time series.
%     An (nx2)-matrix.
% thresholds - The thresholds used to signal a crisis. A (2x1)-vector.
% is_crisis_series - A binary (nx1)-vector. 1 signals that there is a
%   crisis.
% horizon - In the paper it is set 1 to 3 years.
```

G.30   Function `stress_scenario_selection`

```
function forecast_error = stress_scenario_selection(gdp_growth_series)
%
% Based on paper: "Macro Stress Tests and Crises: What can we learn? R.
% Alfaro, M. Drehman, 2009.
% Calculates the worst forecast error in an AR process which is
% subsequently used as a stress scenario.
%
% PARAMETERS:
% gdp_growth_series - The gdp growth series as an (nx1)-vector.
```

G.31   Function `fit_ar_model`

```
function [regressor_coefficients, order] = fit_ar_model(y_series, max_order)
%
% Fits an AR model to the y_series. It selects the order that is less than
% max_order and minimizes the BIC.
%
% PARAMETERS:
% y_series - An (nx1)-vector that we 'll fit an AR model to.
% max_order - The maximum order of AR model we will try.
```

G.32   Function `systemic_risk_exposures`

```
function res = systemic_risk_exposures(exposures,k)
%
% This is based on the paper "Systemic Risk Exposures: A 10-by-10-by-10
% approach" by D.Duffie.
% For each stress test, for each important institution it gives the
% k companies that the important institution has largest exposures on.
%
```

```
% PARAMETERS:
% exposures - The exposures of the important institutions under different
%   stress tests (gains or losses). An (nxpxm)-array. There are m stress
%   tests and n important institutions. For each test for each important
%   institution there are exposures to p companies.
```

<div style="border: 1px solid black;">

Classification of Models

The research literature on the measurement of systemic financial risk is large and growing rapidly. Like the problem it addresses, the set of approaches adopted is wide-ranging and complex. To fully comprehend such a diverse ecosystem, a taxonomy of the various techniques is often useful. This article, for example, is organized primarily around the data requirements of the various approaches. However, the following is another possible categorization, which focuses, instead, on the systemic features examined and the methods used:

- *Illiquidity measures* $\ll$**i**$\gg$ – models focusing on effective supply and demand for securities, mostly in the over-the-counter markets, and typically applying regression techniques.

- *Probability-distribution measures* $\ll$**p**$\gg$ – these apply a more traditional risk-management approach, which assumes that risk is driven by a stable and exogenous data-generating process.

- *Contingent-claim and default measures* $\ll$**c**$\gg$ – models with a salient focus on asymmetric and/or nonlinear payoffs, including the asymmetries arising from the limited-liability structure of financial firms.

- *Network analysis measures* $\ll$**n**$\gg$ – these concentrate attention on the relationships (typically contractual) between participants in the system, and frequently apply graph-theoretic techniques.

- *Macroeconomic measures* $\ll$**m**$\gg$ – models focusing on high-level features of the financial system and/or macro economy, and typically applying time-series techniques, including forecasting.

The tags given here in guillemets also appear in the list of references below to indicate the approach taken by the individual research works. There remains here a large residual category of other measures, denoted by the tag $\ll$**o**$\gg$, as any such categorization is necessarily approximate and incomplete. No single classification scheme will be appropriate for all purposes.

</div>

References

Abbe, E., A. Khandani, and A. W. Lo, 2011, "Privacy-Preserving Methods for Sharing Financial Risk Exposures," working paper, MIT Laboratory for Financial Engineering.

Acharya, V., L. Pedersen, T. Philippon, and M. Richardson, 2010, "Measuring Systemic Risk," working paper, New York University.

Acharya, V., and M. Richardson (eds.), 2009, *Restoring Financial Stability: How to Repair a Failed System*. Wiley, New York.

Adalid, R., and C. Detken, 2007, "Liquidity Shocks and Asset Price Boom/Bust Cycles," ECB Working Paper 732, European Central Bank.

Adrian, T., and M. Brunnermeier, 2010, "CoVaR," Staff Report 348, Federal Reserve Bank of New York.

Adrian, T., and H. S. Shin, 2009, "The shadow banking system: implications for financial regulation," *Financial Stability Review*, 19, 1–10.

———, 2010, "Liquidity and leverage," *Journal of Financial Intermediation*, 19(3), 418–437.

Aikman, D., P. Alessandri, B. Eklund, P. Gai, S. Kapadia, E. Martin, N. Mora, G. Sterne, and M. Willison, 2010, "Funding Liquidity Risk in a Quantitative Model of Systemic Stability," in *Financial Stability, Monetary Policy, and Central Banking*, ed. by R. A. Alfaro. Central Bank of Chile, 12th Annual Conference of the Central Bank of Chile, November 6-7, 2008.

Alessi, L., and C. Detken, 2009, "Real time early warning indicators for costly asset price boom/bust cycles: A role for global liquidity," ECB Working Paper 1039, European Central Bank.

Alexander, L., 2010, "Opening Remarks," working paper, Measuring Systemic Risk: A Conference Sponsored by the Milton Friedman Institute, the Chicago Fed, and the New York Fed.

Alfaro, R., and M. Drehmann, 2009, "Macro stress tests and crises: what can we learn?," *BIS Quarterly Review*, pp. 29–41.

Allen, F., and D. Gale, 2000, "Financial Contagion," *Journal of Political Economy*, 108(1), 1–33.

Amihud, Y., 2002, "Illiquidity and stock returns: Cross-section and time-series effects," *Journal of Financial Markets*, 5, 31–56.

Ang, A., and G. Bekaert, 2002, "International asset allocation with regime shifts," *Review of Financial Studies*, 15(4), 1137–1187.

Ang, A., and J. Chen, 2002, "Asymmetric correlations of equity portfolios," *Journal of Financial Economics*, 63(3), 443–494.

Aragon, G., and P. Strahan, 2009, "Hedge funds as liquidity providers: Evidence from the Lehman bankruptcy," NBER Working Paper Series 15336, National Bureau of Economic Research.

Ashcraft, A., and T. Schuermann, 2008, "Understanding the Securitization of Subprime Mortgage Credit," Federal Reserve Bank of New York Staff Reports 318, Federal Reserve Bank of New York.

Axelrod, R., 1997, *The Complexity of Cooperation: Agent-Based Models of Competition and Collaboration.* Princeton University Press, Princeton, NJ.

Bank of England, B., 2009, "The Role of Macroprudential Policy," Discussion paper, Bank of England.

Basel Committee on Banking Supervision, 2010, "Countercyclical capital buffer proposal," Consultative document, Bank for International Settlements.

———, 2011, "Global systemically important banks: Assessment methodology and the additional loss absorbency requirement," Consultative document, Bank for International Settlements.

Basurto, M., and P. Padilla, 2006, "Portfolio Credit Risk and Macroeconomic Shocks: Applications to Stress Testing Under Data-Restricted Environments," IMF Working Paper WP/06/283, IMF.

Benston, G. J., and G. G. Kaufman, 1997, "FDICIA After Five Years," *The Journal of Economic Perspectives*, 11(3), 139–158.

Bignon, V., M. Flandreau, and S. Ugolini, 2009, "Bagehot for beginners: The making of lending of last resort operations in the mid-19th century," Norges Bank working paper 2009/22, Norges Bank.

Billio, M., and S. Di Sanzo, 2006, "Granger-causality in Markov switching models," Dept. of Economics Research Paper Series 20WP, University Ca' Foscari of Venice.

Billio, M., M. Getmansky, A. W. Lo, and L. Pelizzon, 2010, "Econometric measures of systemic risk in the finance and insurance sectors," NBER Working Paper 16223, NBER.

Black, F., and M. Scholes, 1973, "The pricing of options and corporate liabilities," *Journal of Political Economy*, 81(3), 637–654.

Bollerslev, T., 1986, "Generalized autoregressive conditional heteroskedasticity," *Journal of Econometrics*, 31(3), 307–327.

Boot, A., and A. Thakor, 1993, "Self-Interested Bank Regulation," *American Economic Review*, 83(2), 206–212.

Bordo, M., 1990, "The Lender of Last Resort: Alternative Views and Historical Experience," *Federal Reserve Bank of Richmond Economic Review*, 1990, 18–29.

Borio, C., 2009, "The macroprudential approach to regulation and supervision," working paper, VoxEU.org, 14 April 2009.

———, 2010, "Implementing a macroprudential framework: blending boldness and realism," working paper, Bank for International Settlements, Keynote address for the BIS-HKMA research conference, Honk Kong SAR, 5-6 July 2010.

Borio, C., and M. Drehmann, 2008, "Towards an operational framework for financial stability: "Fuzzy" measurement and its consequences," *12th Annual Conference of Banco de Chile on Financial Stability, Monetary Policy and Central Banking.*

Borio, C., and M. Drehmann, 2009a, "Assessing the risk of banking crises – revisited," *BIS Quarterly Review*, 2009(2), 29–46.

———, 2009b, "Towards an operational framework for financial stability: 'fuzzy' measurement and its consequences," BIS Working Papers 284, Bank for International Settlements.

Borio, C., and P. Lowe, 2004, "Securing sustainable price stability: should credit come back from the wilderness?," BIS Working Paper 157, Bank for International Settlements.

Bossaerts, P., 2009, "What Decision Neuroscience Teaches Us About Financial Decision Making," *Annual Review of Financial Economics*, 1, 383–404.

Bottega, J. A., and L. F. Powell, 2010, "Creating a Linchpin for Financial Data: The Need for a Legal Entity Identifier," working paper, Board of Governors of the Federal Reserve.

Bouchaud, J., J. D. Farmer, and F. Lillo, 2009, "How Markets Slowly Digest Changes in Supply and Demand," in *Handbook of Financial Markets: Dynamics and Evolution*, ed. by H. Thorsten, and K. chenk Hoppe. Elsevier, New York.

Boyd, J., and M. Gertler, 1994, "Are Banks Dead? Or Are the Reports Greatly Exaggerated?," *Federal Reserve Bank of Minneapolis Quarterly Review*, 18(3), 2–23.

Boyson, N. M., C. W. Stahel, and R. M. Stulz, 2010, "Hedge Fund Contagion and Liquidity Shocks," *Journal of Finance*, 65(5), 1789–1816.

Breiman, L., J. Friedman, R. Olshen, and C. Stone, 1984, *Classification and Regression Trees*. Wadsworth and Brooks Cole Advanced Books and Software, Pacific Grove, CA.

Breuer, T., M. Jandačka, K. Rheinberger, and M. Summer, 2009, "How to Find Plausible, Severe and Useful Stress Scenarios," *International Journal of Central Banking*, 5(3), 205–224, Bank for International Settlements.

Brockwell, P., and R. Davis, 1991, *Time Series: Theory and Methods*. Springer, New York, NY.

Brown, C. O., and I. S. Din, 2011, "Too Many to Fail? Evidence of Regulatory Forbearance When the Banking Sector Is Weak," *Review of Financial Studies*, 24(4), 1378–1405.

Brunnermeier, M., and L. Pedersen, 2009, "Market liquidity and funding liquidity," *Review of Financial Studies*, 22(6), 2201–2238.

Brunnermeier, M. K., A. Crockett, C. A. Goodhart, A. D. Persaud, and H. S. Shin, 2009, "The Fundamental Principles of Financial Regulation," Geneva Reports on the World Economy 11, International Center for Monetary and Banking Studies.

Brunnermeier, M. K., G. Gorton, and A. Krishnamurthy, 2010, "Risk Topography," working paper, Princeton University.

Caballero, R. J., 2009, "The 'Other' Imbalance and the Financial Crisis," MIT Department of Economics Working Paper No. 09-32, Massachusetts Institute of Technology.

Capuano, C., 2008, "The option-iPoD. The Probability of Default Implied by Option Prices Based on Entropy," IMF Working Paper 08/194, International Monetary Fund.

Caruana, J., 2010a, "Financial Stability: Ten Questions and about Seven Answers," in *Reserve Bank of Australia 50th Anniversary Symposium*.

——— , 2010b, "Macroprudential policy: could it have been different this time?," working paper, Bank for International Settlements, Peoples Bank of China seminar on macroprudential policy in cooperation with the International Monetary Fund: Shanghai, Monday 18 October 2010.

Chan, N., M. Getmansky, S. M. Haas, and A. W. Lo, 2006a, "Do Hedge Funds Increase Systemic Risk?," *Federal Reserve Bank of Atlanta Economic Review*, 91(4), 49–80.

——— , 2006b, "Systemic risk and hedge funds," in *The Risks of Financial Institutions*, ed. by M. Carey, and R. Stulz. University of Chicago Press, Chicago, IL, pp. 235–330.

Chan-Lau, J., 2009, "Co-risk measures to assess systemic financial linkages," Imf working paper, Internanational Monetary Fund.

Chan-Lau, J., M. Espinosa, and J. Sole, 2009, "On the use of network analysis to assess systemic financial linkages," Imf working paper, International Monetary Fund.

Clement, P., 2010, "The term "macroprudential": origins and evolution," *BIS Quarterly Review*, 2010, 59–67.

Cover, T., and J. Thomas, 2006, *Elements of Information Theory*. Wiley-Interscience, New York, NY.

Cox, J., and M. Rubinstein, 1985, *Options Markets*. Prentice-Hall, Englewood Cliffs, NJ.

Danielsson, J., and H. S. Shin, 2003, "Endogenous Risk," in *Modern Risk Management: A History*. Risk Books, New York.

De Bandt, O., and P. Hartmann, 2000, "Systemic Risk: A Survey," Working Paper 35, European Central Bank.

Demirguc-Kunt, A., E. Kane, and L. Laeven, 2008, "Determinants of Deposit-Insurance Adoption and Design," *Journal of Financial Intermediation*, 17(3), 407–438.

Drehmann, M., 2009, "Macroeconomic stress testing banks: A survey of methodologies," in *Stress Testing the Banking System: Methodologies and Applications*, ed. by M. Quagliariello. Cambridge University Press, Cambridge, UK, pp. 37–67.

Duffie, D., 2010, *How Big Banks Fail and What to Do about It*. Princeton University Press, Princeton, NJ.

———, 2011, "Systemic Risk Exposures A 10-by-10-by-10 Approach," working paper, Stanford University.

Engle, R., 2002, "Dynamic conditional correlation: A simple class of multivariate generalized autoregressive conditional heteroskedasticity models," *Journal of Business and Economic Statistics*, 20, 339–350.

European Central Bank (ECB), 2010, "Financial networks and financial stability," *Financial Stability Review*, 2010, 155–160.

Farmer, J. D., and D. Foley, 2009, "The economy needs agent-based modelling," *Nature*, 460, 685–686.

Feldman, R., and M. Lueck, 2007, "Are Banks Really Dying This Time? An update of Boyd and Gertler.," *The Region*, 2007, 6–51.

Fender, I., and P. McGuire, 2010a, "Bank structure, funding risk and the transmission of shocks across countries: concepts and measurement," *BIS Quarterly Review*, 2010, 63–79.

Fender, I., and P. McGuire, 2010b, "European banks' U.S. dollar funding pressures," *BIS Quarterly Review*, pp. 57–64.

Fielding, E., A. W. Lo, and J. H. Yang, 2011, "The National Transportation Safety Board: A Model for Systemic Risk Management," *Journal of Investment Management*, 9, 18–50.

Financial Stability Board, 2009, "Guidance to Assess the Systemic Importance of Financial Institutions, Markets and Instruments: Initial Considerations," Report to g20 finance ministers and governors, Financial Stability Board.

———, 2011, "Shadow Banking: Scoping the Issues, A Background Note of the Financial Stability Board," working paper, Financial Stability Board.

Financial Stability Board and International Monetary Fund, 2010, "The Financial Crisis and Information Gaps Progress Report Action Plans and Timetables," working paper, FSB.

Flood, M., 1992, "The Great Deposit Insurance Debate," *Federal Reserve Bank of St. Louis Review*, 74(4), 51–77.

Freixas, X., B. M. Parigi, and J.-C. Rochet, 2000, "Systemic Risk, Interbank Relations, and Liquidity Provision by the Central Bank," *Journal of Money, Credit and Banking*, 32(3), 611–638, What Should Central Banks Do? A conference sponsored by the Federal Reserve Bank of Cleveland, Oct. 27-29, 1999.

Freund, Y., and R. Shapire, 1996, "Experiments with a new boosting algorithm," *Proceedings of the Thirteenth International Conference on Machine Learning*, pp. 148–156.

Geanakoplos, J., 2010, "Solving the Present Crisis and Managing the Leverage Cycle," *Federal Reserve Bank of New York Economic Policy Review*, 16(1), 101–131.

Getmansky, M., A. W. Lo, and I. Makarov, 2004, "An econometric model of serial correlation and illiquidity in hedge fund returns," *Journal of Financial Economics*, 74(3), 529–609.

Giesecke, K., and B. Kim, 2009, "Risk analysis of collateralized debt obligations," Working Paper.

Glasserman, P., and J. Li, 2005, "Importance sampling for portfolio credit risk," *Management Science*, 51, 1643–1656.

Gorton, G., and A. Metrick, 2010, "Regulating the Shadow Banking System," *Brookings Papers on Economic Activity*, 2010, 261–312.

Gray, D., and A. Jobst, 2010, "Systemic CCA – A Model Approach to Systemic Risk," working paper, International Monetary Fund, Paper presented at conference sponsored by the Deutsche Bundesbank and Technische Universitaet Dresden, 28-29 October 2010.

Gray, D., and A. Jobst, 2011, "Systemic contingent claims analysis (Systemic CCA)—Estimating potential losses and implicit government guarantees to the financial sector," Imf working paper, International Monetary Fund.

Group of Ten, 2001, "Report on Consolidation in the Financial Sector: Chapter III. Effects of consolidation on financial risk," working paper, International Monetary Fund.

Hamilton, J. D., 1994, *Time Series Analysis*. Princeton University Press, Princeton, NJ.

Hanson, S. G., A. K. Kashyap, and J. C. Stein, 2011, "A Macroprudential Approach to Financial Regulation," *Journal of Economic Perspectives*, 25(1), 3–28.

Hasbrouck, J., 2007, *Empirical Market Microstructure: The Institutions, Economics, and Econometrics of Securities Trading*. Oxford University Press, New York, NY.

Hirtle, B., T. Schuermann, and K. Stiroh, 2009, "Macroprudential Supervision of Financial Institutions: Lessons from the SCAP," Staff Report No. 409, Federal Reserve Bank of New York.

Hu, X., J. Pan, and J. Wang, 2010, "Noise as Information for Illiquidity," working paper, Massachusetts Institute of Technology.

Huang, X., H. Zhou, and H. Zhu, 2009a, "Assessing the Systemic Risk of a Heterogeneous Portfolio of Banks During the Recent Financial Crisis," Federal Reserve Board Finance and Economics Discussion Series 2009-44, Board of Governors of the Federal Reserve.

——— , 2009b, "A framework for assessing the systemic risk of major financial institutions," working paper, University of Oklahoma.

Huizinga, H., and L. Laeven, 2010, "Bank Valuation and Regulatory Forbearance During a Financial Crisis," working paper, Centre for Economic Policy Research (CEPR).

Hull, J., 2000, *Options, Futures, and Other Derivatives*. Prentice-Hall, Upper Saddle River, NJ.

International Monetary Fund, 2009a, "Assessing the Systemic Implications of Financial Linkages," *Global Financial Stability Review*, Apr09, 73–110.

———, 2009b, "Global Financial Stability Report: Responding to the Financial Crisis and Measuring Systemic Risks," working paper, IMF.

———, 2011, "Global Financial Stability Report: Grappling with Crisis Legacies," working paper, IMF.

Jolliffe, I., 2002, *Principal Component Analysis*. Springer, New York, NY.

Kahneman, D., and A. Tversky, 1979, "Prospect Theory: An Analysis of Decision Under Risk," *Econometrica*, 47, 263–291.

Kaminsky, G., S. Lizondo, and C. Reinhart, 1998, "Leading indicators of currency crisis," *IMF Staff Paper*, 1.

Kane, E. J., 1989, *The S&L Insurance Mess: How Did It Happen?* Urban Institute Press, Washington, D.C.

Kapadia, S., M. Drehmann, J. Elliott, and G. Sterne, 2009, "Liquidity Risk, Cash Flow Constraints, and Systemic Feedbacks," working paper, Bank of England.

Khandani, A. E., A. J. Kim, and A. W. Lo, 2010, "Consumer Credit Risk Models via Machine-Learning Algorithms," *Journal of Banking and Finance*, 34(11), 2767–2787.

Khandani, A. E., and A. W. Lo, 2007, "What happened to the quants in August 2007?," *Journal of Investment Management*, 5(4), 5–54.

———, 2011, "What Happened To The Quants In August 2007?: Evidence from Factors and Transactions Data," *Journal of Financial Markets*, 14(1), 1–46.

Khandani, A. E., A. W. Lo, and R. C. Merton, 2009, "Systemic Risk and the Refinancing Ratchet Effect," MIT Sloan School Working Paper 4750-09, MIT.

King, M. R., and P. Maier, 2009, "Hedge Funds and Financial Stability: Regulating Prime Brokers Will Mitigate Systemic Risks," *Journal of Financial Stability*, 5(3), 283–297.

Klaus, B., and B. Rzepkowski, 2009, "Hedge funds and brokers," Goethe university working paper, Goethe University.

Kocherlakota, N., 2010, "Modern Macroeconomic Models as Tools for Economic Policy," *The Region*, (May), 5–21.

Koenker, R., 2005, *Quantile Regression*. Cambridge University Press, Cambridge, UK.

Koenker, R., and K. Hallock, 2001, "Quantile regression," *Journal of Economic Perspectives*, 15, 143–156.

Kritzman, M., and Y. Li, 2010, "Skulls, Financial Turbulence, and Risk Management," *Financial Analysts Journal*, 66(5), 30–41.

Kritzman, M., Y. Li, S. Page, and R. Rigobon, 2010, "Principal Components as a Measure of Systemic Risk," Revere Street Working Paper Series: Financial Economics 272-28, Revere Street Working Paper Series.

Kullback, S., and R. Leibler, 1951, "On information and sufficiency," *The Annals of Mathematical Statistics*, 22, 449–470.

Kyle, A., 1985, "Continuous auctions and insider trading," *Econometrica*, 53, 1315–1335.

Laux, C., and C. Leuz, 2010, "Did Fair-Value Accounting Contribute to the Financial Crisis?," *Journal of Economic Perspectives*, 24(1), 93–118.

Lee, S., 2010, "Measuring systemic funding liquidity risk in the interbank foreign currency ending market," Working Paper 418, Bank of Korea Institute for Monetary and Economic Research.

Lerner, J., 2002, "Where Does *State Street* Lead?: A First Look at Financial Patents, 1971–2000," *Journal of Finance*, 57, 901–930.

Ljung, G., and G. Box, 1978, "On a measure of lack of fit in time series models," *Biometrika*, 65, 297–303.

Lo, A. W., 2011, "Fear, Greed, and Financial Crises: A Cognitive Neurosciences Perspective," in *Handbook on Systemic Risk*, ed. by J. Fouque, and J. Langsam. Cambridge University Press, Cambridge, UK.

Lo, A. W., and C. MacKinlay, 1988, "Stock market prices do not follow random walks: Evidence from a simple specification test," *Review of Financial Studies*, 1, 41–66.

———— , 1990a, "When are contrarian profits due to stock market overreaction?," *Review of Financial Studies*, 3, 175–205.

Lo, A. W., and W. K. Newey, 2011, "Temporal Averaging and Nonstationarity," working paper, MIT Laboratory for Financial Engineering.

Longstaff, F., 2004, "The flight-to-liquidity premium in U.S. Treasury bond prices," *Journal of Business*, 77, 511–525.

Loutskina, E., and P. E. Strahan, 2009, "Securitization and the Declining Impact of Bank Finance on Loan Supply: Evidence from Mortgage Originations," *Journal of Finance*, 64(2), 861–889.

Lucas, D. (ed.), 2010, *Measuring and Managing Federal Financial Risk*. University of Chicago Press, Chicago, IL.

Lucas, R. E., 1976, "Econometric Policy Evaluation: A Critique," in *The Phillips Curve and Labor Markets: Carnegie-Rochester Conference Series on Public Policy 1*, ed. by K. Brunner, and A. Meltzer. Elsevier, New York, pp. 19–46.

Mantegna, R., and E. Stanley, 2000, *An Introduction to Econophysics: Correlations and Complexity in Finance*. Cambridge University Press, Cambridge, UK.

Mayhew, S., 1995, "Implied volatility," *Financial Analysts Journal*, 51(4), 8–20.

McCulley, P., 2010, "After the Crisis: Planning a New Financial Structure," Global central bank focus, PIMCO, Based on Comments Before the 19th Annual Hyman Minsky Conference on the State of the U.S. and World Economies, April 15, 2010.

Merton, P., 1937, "On the generalised distance in statistics," *Proceedings of the National Institute of Sciences in India*, 2(1), 49–55.

Merton, R., 1973, "Theory of rational option pricing," *Journal of Economics and Management Science*, 4, 141–183.

Merton, R., and Z. Bodie, 1993, "Deposit Insurance Reform: A Functional Approach," *Carnegie-Rochester Conference Series on Public Policy*, 38, 1–34.

Minsky, H., 1982, *Can "It" Happen Again?: Essays on Instability and Finance*. M.E. Sharpe, Armonk, NY.

Mishkin, F. S., 2007, "Systemic Risk and the International Lender of Last Resort," working paper, Board of Governors of the Federal Reserve, Speech delivered at the Tenth Annual International Banking Conference, Federal Reserve Bank of Chicago, September 28, 2007.

Moussa, A., 2011, "Contagion and Systemic Risk in Financial Networks," Ph.D. thesis, Columbia University.

Nier, E., J. Yang, T. Yorulmazer, and A. Alentorn, 2008, "Network models and financial stability," Working Paper 346, Bank of England.

Nijskens, R., and W. Wagner, 2011, "Credit Risk Transfer Activities and Systemic Risk: How Banks Became Less Risky Individually But Posed Greater Risks to the Financial System at the Same Time," *Journal of Banking & Finance*.

Office of Financial Research, 2010, "Statement on Legal Entity Identification for Financial Contracts," *Federal Register, (229), 30 November 2010,*, 75(229), 74146–74148, 30 November 2010.

———, 2011, "Office of Financial Research Issues Statement on Progress to Date and Next Steps Forward in the Global Initiative to Establish a Legal Entity Identifier (LEI),," Press release, OFR, 12 August 2011.

Pastor, L., and R. Stambaugh, 2003, "Liquidity risk and expected stock returns," *Journal of Political Economy*, 111, 642–685.

Peltzman, S., 1975, "The Effects of Automobile Safety Regulation," *Journal of Political Economy*, 83, 677–725.

Pojarliev, M., and R. M. Levich, 2008, "Do professional currency managers beat the benchmark?," *Financial Analysts Journal*, 64(5), 18–32.

——— , 2011, "Detecting Crowded Trades in Currency Funds," *Financial Analysts Journal Volume Number 1*, 67(1), 26–39.

Pozsar, Z., T. Adrian, A. Ashcraft, and H. Boesky, 2010, "Shadow Banking," Staff Reports 458, Federal Reserve Bank of New York.

Reinhart, C. M., and K. Rogoff, 2008, "This Time is Different: A Panoramic View of Eight Centuries of Financial Crises," NBER Working Paper 13882, NBER.

——— , 2009, *This Time Is Different: Eight Centuries of Financial Folly*. Princeton University Press, Princeton, NJ.

Ricks, M., 2010, "Shadow Banking and Financial Regulation," Columbia law and economics working paper no. 370, Harvard Law School.

Rochet, J.-C., and X. Vives, 2004, "Coordination Failures and the Lender of Last Resort: Was Bagehot Right after All?," *Journal of the European Economic Association*, 2(6), 1116–1147.

Rosengren, E. S., 2010, "Asset Bubbles and Systemic Risk," working paper, Federal Reserve Bank of Boston, Speech delivered at the Global Interdependence Center's Conference on "Financial Interdependence in the World's Post-Crisis Capital Markets", Philadelphia, March 3, 2010.

Sapra, H., 2008, "Do accounting measurement regimes matter? A discussion of mark-to-market accounting and liquidity pricing," *Journal of Accounting and Economics*, 45(2-3), 379–387.

Schwarz, G., 1978, "Estimating the dimension of a model," *Annals of Statistics*, 6(2), 461–464.

Segoviano, M., 2006, "The consistent information multivariate density optimizing methodology," Financial Markets Group Discussion Paper 557, London School of Economics.

Segoviano, M. A., and C. Goodhart, 2009, "Banking stability measures," Financial Markets Group, Discussion paper 627, London School of Economics and Political Science.

Svennson, L., 1994, "Estimating and interpreting forward interest rates: Sweden 1992–1994," NBER Working Paper 4871, National Bureau of Economic Research.

Tarashev, N., and H. Zhu, 2008, "Specification and calibration errors in measure of portfolio credit risk: The case of the ASRF model," *Interational Journal of Central Banking*, 4, 129–174.

Upper, C., 2007, "Using counterfactual simulations to assess the danger of contagion in interbank markets," BIS Working Paper 234, Bank for International Settlements.

Valukas, A., 2010, "Report of Anton R. Valukas, Examiner: Volume 3 Of 9, Section III.A.4: Repo 105," working paper, United States Bankruptcy Court Southern District Of New York, In Re Lehman Brothers Holdings Inc., et al., Debtors. Chapter 11 Case No. 08-13555.

Vasicek, O., 1991, "Limiting loan loss probability distribution," Kmv working paper, KMV.

Woolridge, J., 2002, *Econometric Analysis of Cross Section Panel Data*. The MIT Press, Cambridge, MA.